Mechanisms behind Collective Social Phenomena

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Paul Dirac's speech at the Nobel Banquet, 1933.

"There is [...] *a great similarity between the problems provided by the mysterious behavior of the atom and those provided by the present* [...] *paradoxes confronting the world. In both cases one is given a great many facts which are expressible with numbers, and one has to find the underlying principles.* ***The methods of theoretical physics should be applicable to all those branches of thought in which the essential features are expressible with numbers."***

D. Watts, Everything is obvious. How common sense fails, 2011

Micro-macro: How do we get from the micro choices of individuals to the macro phenomena of the social world?

Something like the micro-macro problem comes up in every realm of science, often under the label of "emergence": How is it, for example that one can lump together a collection of atoms and somehow get a molecule? How is that one can lump together a collection of molecules and somehow get amino acids? How is it that one can lump together a collection of aminoacids and other chemicals and somehow get a living cell? How is that one can lump together a collection of living cells and somehow get complex organs like the brain? And how is that one can lump together a collection of organs and somehow get a sentient being that wonders about its eternal self?

Seen in this light, sociology is merely at the tip of the pyramid of complexity that begins with subatomic particles and ends with global society. And a each level of the pyramid, we have essentially the same problem-how do we get from one "scale" of reality to next?

(Emergence: P.W.Anderson, More is different, Science (1972))

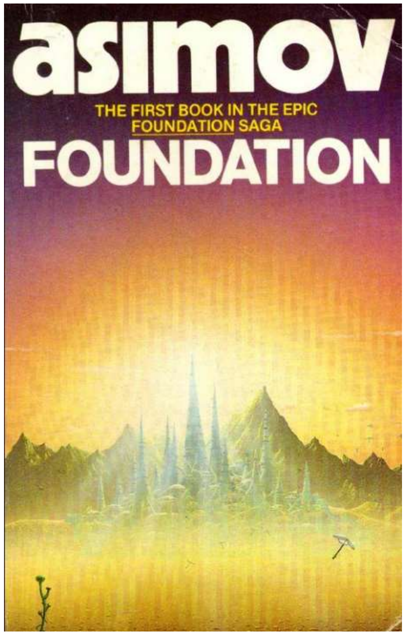
A Definition:

A SYSTEM HAS EMERGENT PROPERTIES WHEN AN EFFECTIVE THEORY OF THE SYSTEM AT SOME SCALE OR LEVEL OF ORGANIZATION, IS QUALITATIVELY DIFFERENT FROM THE LOWER-LEVEL THEORY

SOCIETY AS AN EMERGENT PHENOMENA

Individual → ***Society***

There is no such thing as society: there are individual men and women

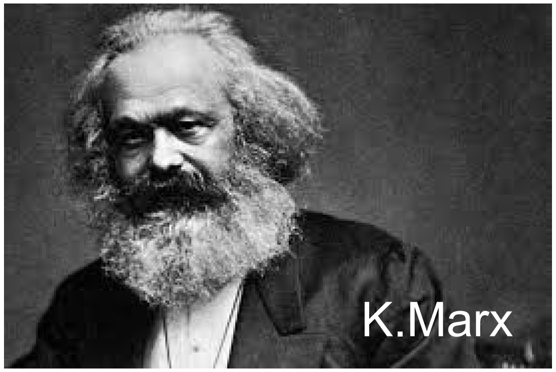


Psycohistory: H. Seldon

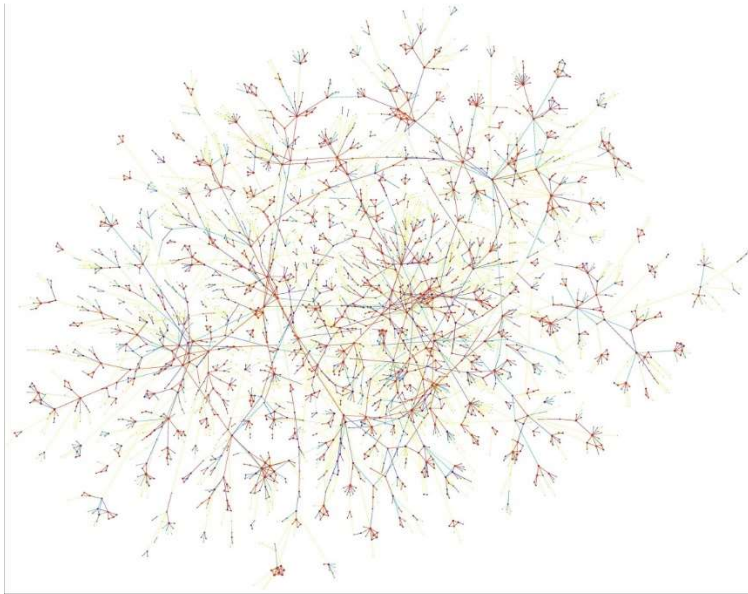
EMERGENCE IS NOT STATISTICS!!

- -Simple problems
- -Statistical Problems
- -Organized Complexity

*W. Weaver, Science and Complexity, American Scientist **36**, 536 (1948)*



Society is not made up of human beings, but constructed in terms of their communications



Agent characterization: state

Binary +1, -1 ; Continuous [0,1]; Vector

Strategy and Pay-off

Interaction rules among agents

Interaction force ⟷ Social mechanism

Pairwise interaction (two body /multiple collisions)
Higher order interactions

Network of interactions: Who interacts with whom?

Activity patterns: When interactions occur

Interaction rules

CONSENSUS: When and how the dynamics of a set of interacting units (agents) that can choose among several **options** leads to a **consensus** in one of these options, or when a state with several **coexisting** options prevails.

CONTAGION: When and how a contagious entity propagates from a seed to a whole system of interacting agents?

CONSENSUS and CONTAGION PROBLEMS

SOCIAL MECHANISMS

- -Imitation
- -Disease transmission
- -Threshold for social pressure
- -Maximize Pay-off (fitness)

↔

MODELS

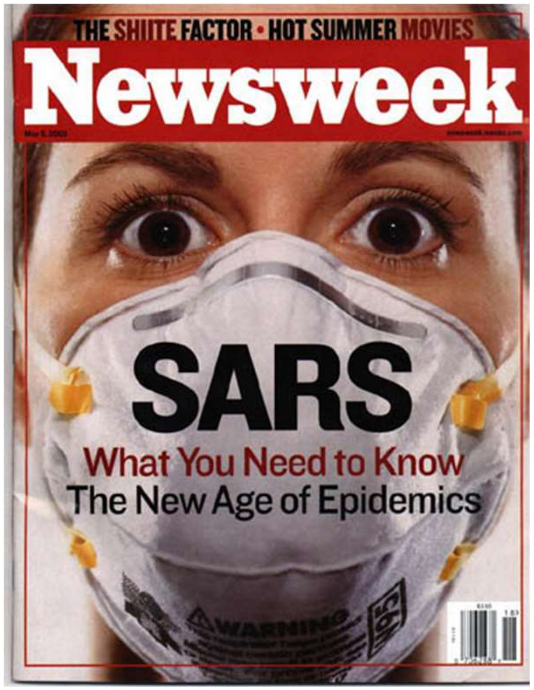
- -VOTER MODEL
- -SIS epidemic model
- -SCHELLING/GRANOVETTER'S MODEL
- -Evolutionary Game Theory: Coordination

Dyadic /two body interaction				Group/nonlinear interaction
	Voter	← Consensus →	Nonlinear Voter Coordination Game	
	SIS	← Contagion →	Threshold models (Schelling/Granovetter)	

How and when a contagious entity propagates from a seed to a whole system?

Two classes of contagion processes: **SIMPLE** and **COMPLEX**

SIMPLE CONTAGION

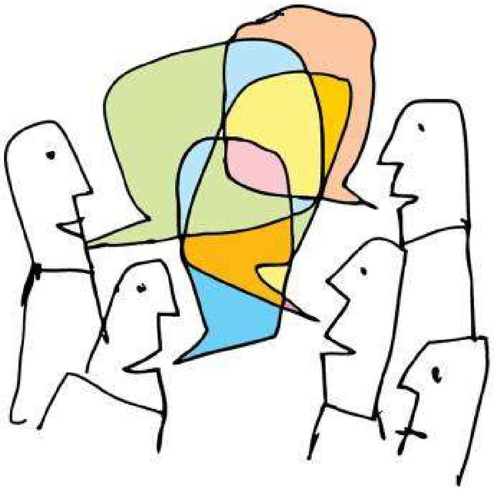


- Disease outbreaks.
- Epidemics
- ….

SIS: Dyadic two body interaction

Continuous transition

COMPLEX CONTAGION

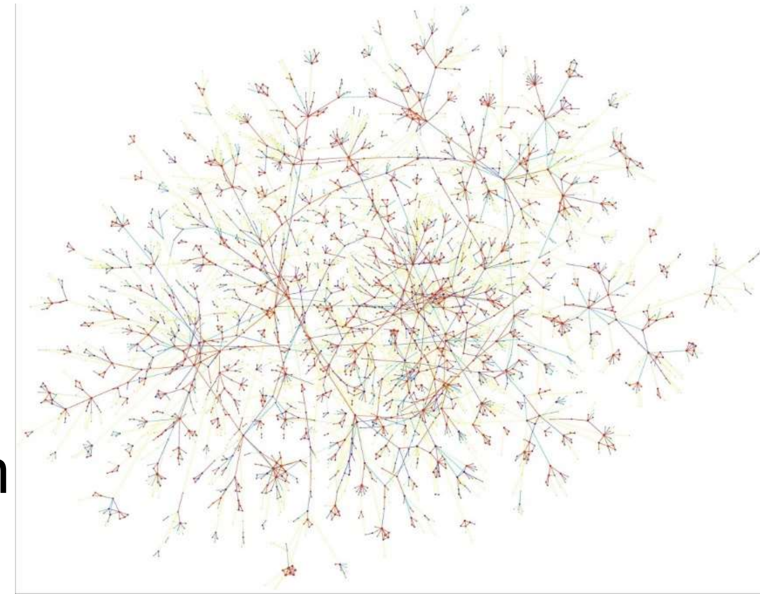


Social contagion of behaviors and innovations

- rumors,
- fads,
- innovations,
- riot paticipation
- information spreading.
- ….

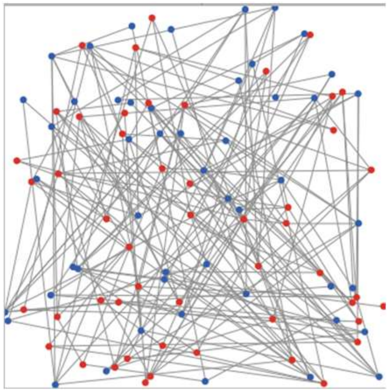
Threshold Models: Group interaction

Cascade discontinuous transition

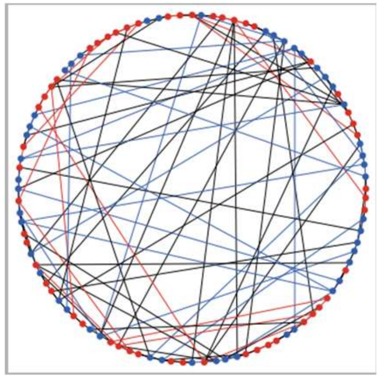


- **Agent characterization: state**
  Binary +1, -1 ; Continuous [0,1]; Vector
  Strategy and Pay-off
- **Interaction rules among agents**
  Interaction force ⟷ Social mechanism
  - Pairwise interaction (two body /multiple collisions)
  - Higher order interactions
- **Network of interactions: Who interacts with whom?**
  Complex networks: Tie heterogeneity (Degree distribution $P(k)$)
  Small world, Scale free, Community structure, Hypergraphs
  Co-evolution : Ties are not persistent
- **Activity patterns: When interactions occur**

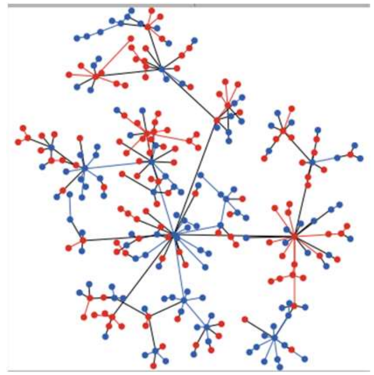
Networks of interaction



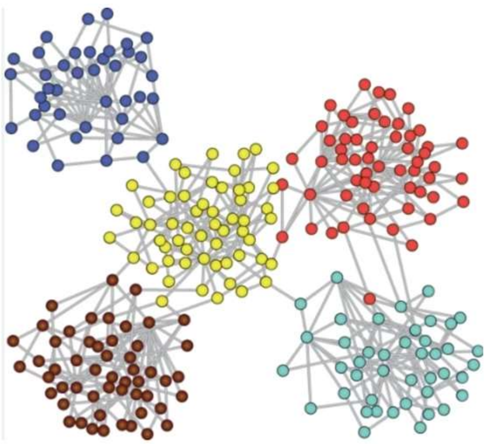
ER Random



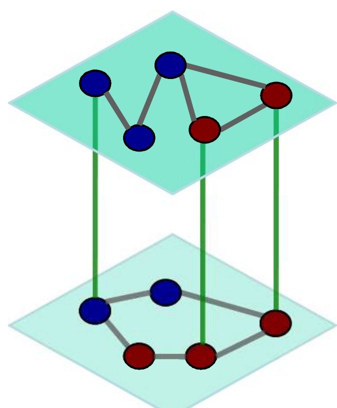
Small World



Scale Free



Community structure



MULTILAYER/ MULTIPLEX

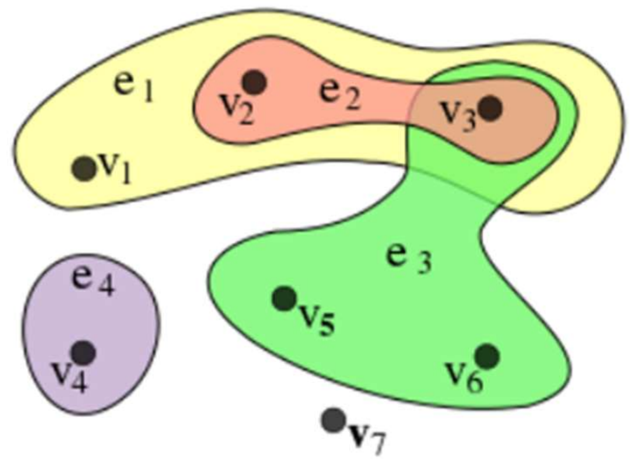
nodes represent agents,
layers represent contexts

Higher Order interactions HYPERGRAPHS

An Hypergraph H is a pair H=(X,E)

X= Set of nodes or vertices

E= Set of nonempty subsets of X: Hyperedges



7 vertices and 4 hyperedges

$$X = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$$

$$E = \{e_1, e_2, e_3, e_4\} = \{\{v_1, v_2, v_3\}, \{v_2, v_3\}, \{v_3, v_5, v_6\}, \{v_4\}\}$$

F. Battiston et al, Phys. Rep. 874, 1 (2020)
G. Bianconi, Higher Order Networks, CUP (2021)

Networks of interaction: CO-EVOLUTION

M. Zimmerman, et al Lecture Notes in Economics and Mathematical Systems 503, (2001)

Dynamics of Networks:

1. Dynamics **OF** network formation: Structure created by individual choices/actions ← **Rightwing view**

2. Dynamics **ON** the network: Actions of individuals constrained by the social network ← **Leftwing view**

3. Co-evolution of agents and network :
Circumstances make men as much as men make circumstances

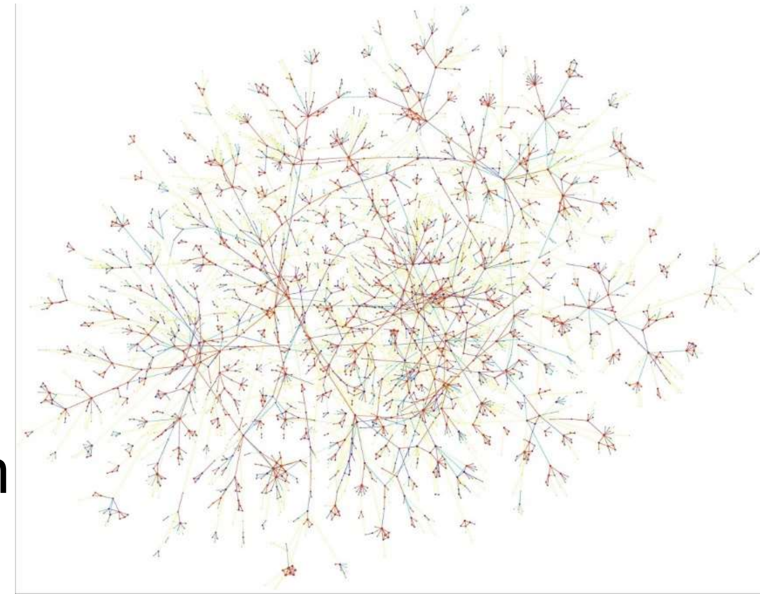
..new research agenda in which the structure of the network is no longer a given but a variable.....explore how a social structure might evolve in tandem with the collective action it makes possible (Macy, Am. J. Soc. 97, 808 (1991))

Final Goal: Understanding *dynamical* processes of group formation and social differentiation: Emergence of social dynamical networks with

-Social structure

-Weak links

-Community structure



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Small world, Scale free, Community structure

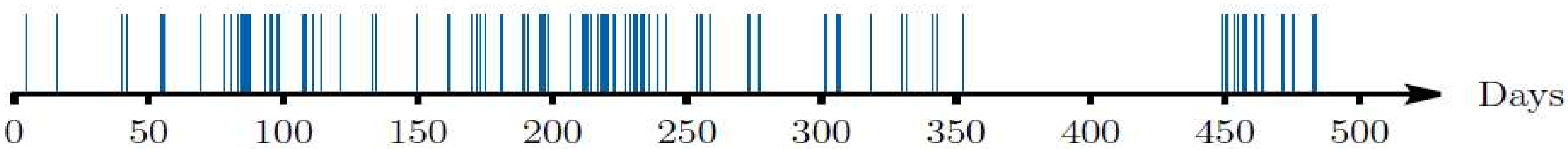
Co-evolution : Ties are not persistent

Activity patterns: When interactions occur

Constant rate or temporal heterogeneity *(Aging)*

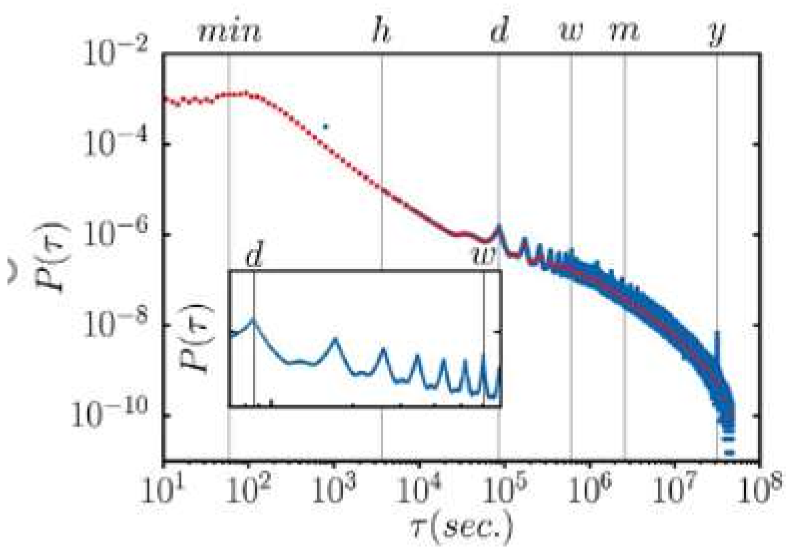
Human Activity Patterns

Artime et al, Sci. Rep. 7:41627(2017)

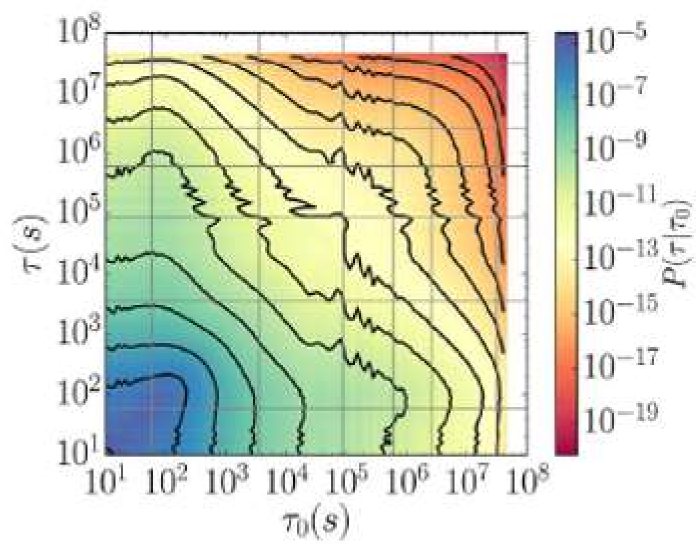


Bursty interaction pattern between two twitter users

Inter-event time (IET) distribution



P(τ) Nonpoissonian



Temporal Correlations P(τ/τ´)

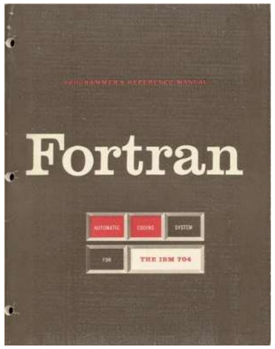
Question:

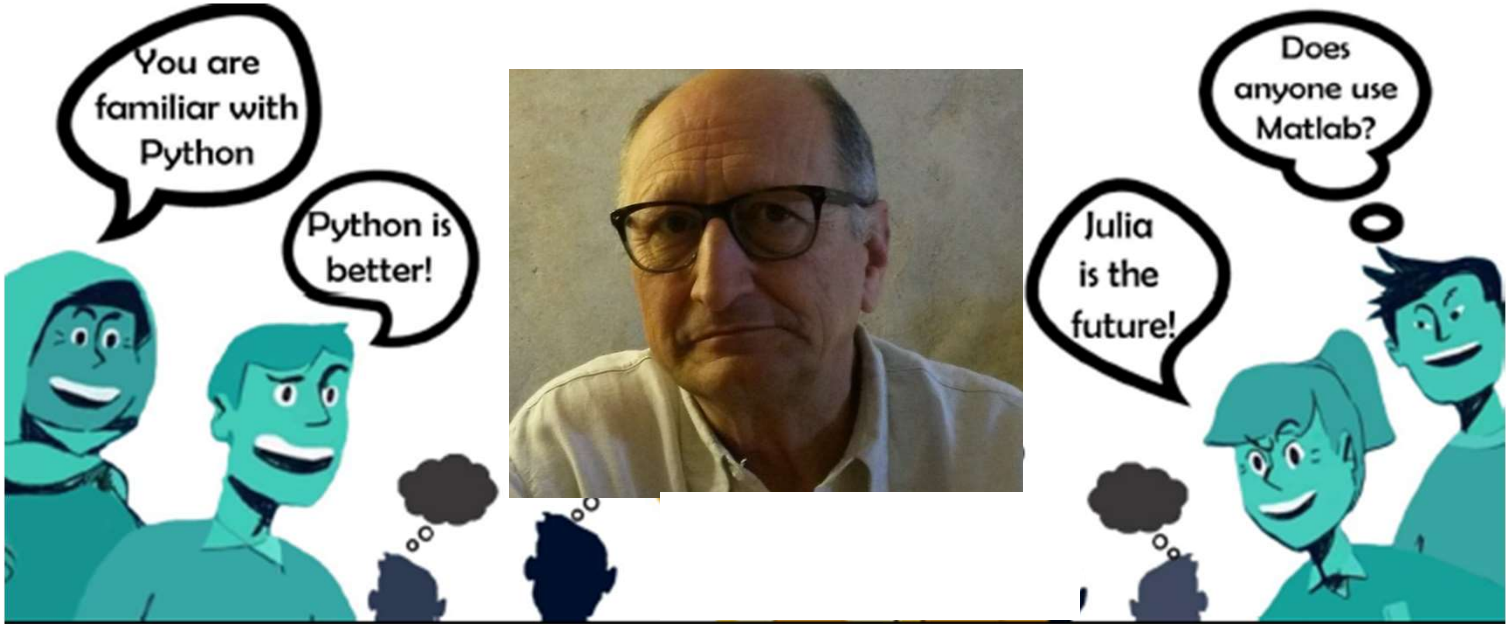
Role of the Timing of Interactions.
How is this modeled in the updating processes?

Standard Monte Carlo simulations assume a constant rate of interaction

AGING: The longer you are in a given state (the longer your persistence time) the smaller is your probability to update

Example: the adoption of programming languages





UNIT OF EXCELLENCE MARÍA DE MAEZTU

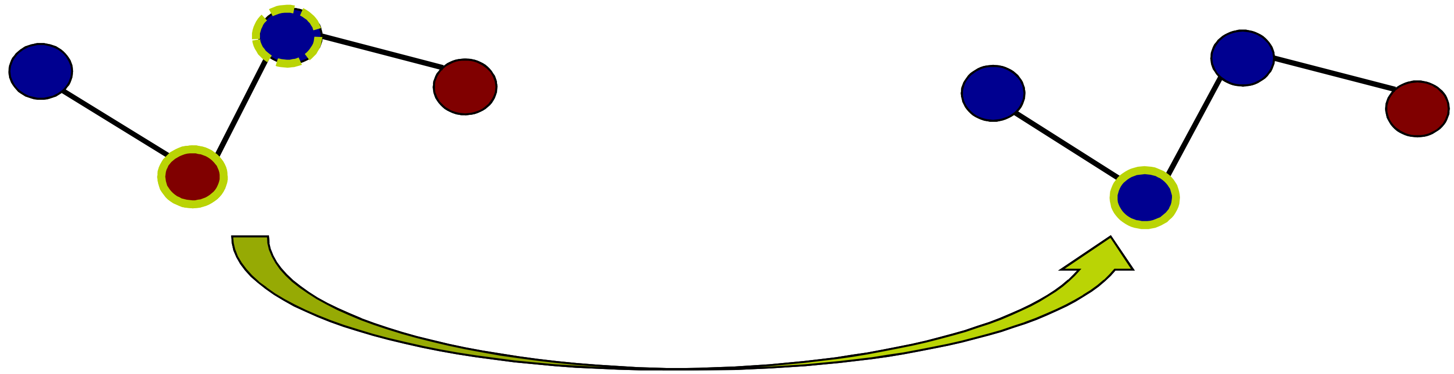
IMITATION

Herding Behavior

Voter Model: Imitation Dynamics

Clifford and Sudbury, Biometrika (1973)
Holley and Ligget, Ann. Probability (1975)

Two options:



Interaction: copy the state of one of your neighbors at random

Question: When and how consensus is reached by imitation?

First lesson: Choice of variables

Average number of nodes in one of the states is conserved

Local variable: ρ Average number of active links (interface density)

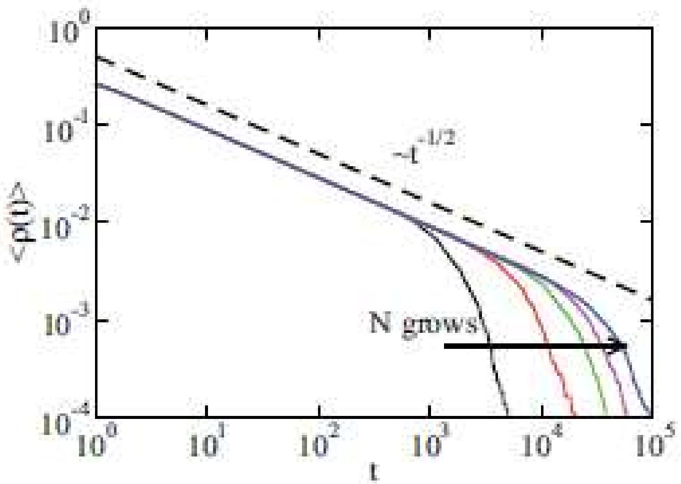
Voter Model in regular networks

$$<\rho>\sim\begin{cases} t^{-1/2}, & d=1 \\ (\ln t)^{-1}, & d=2 \\ \xi - bt^{-d/2}, & d>2 \end{cases} \qquad \tau\sim\begin{cases} N^2, & d=1, \text{ time to reach absorbing state} \\ N\ln N, & d=2, \text{ time to reach absorbing state} \\ N, & d>2, \text{ survival time of metastable state} \end{cases}$$

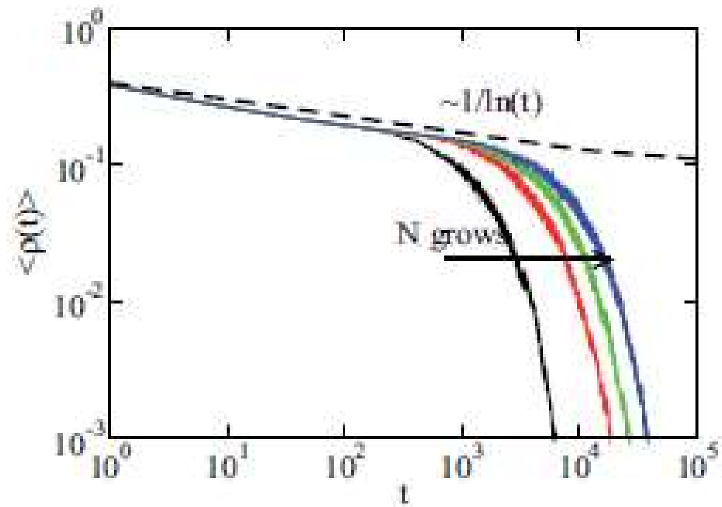
d=1,2: Coarsening/Ordering

Unbounded growth of domains of absorbing states

d=1



d=2



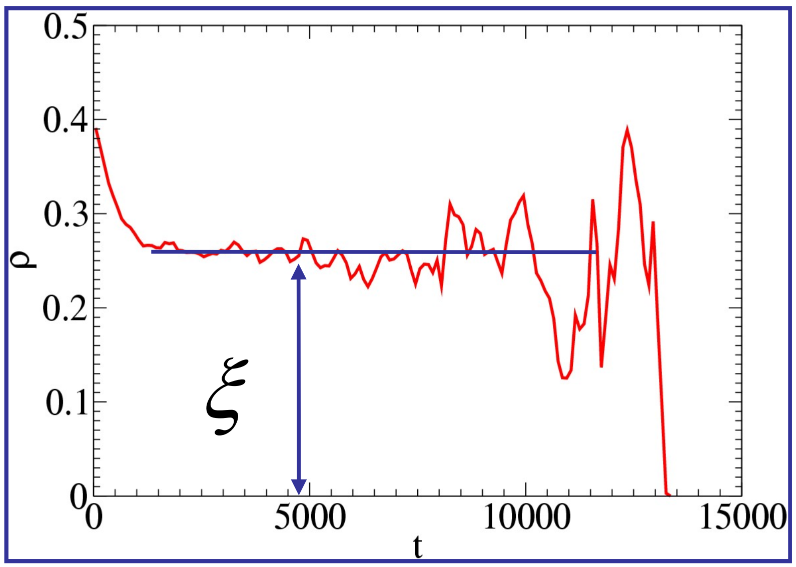
Regular d>2, and complex networks: Random, Small World, Scale Free Networks,…

$< \rho > \sim \xi$

$\tau(N) \approx \mathrm{N}$, survival time of metastable state

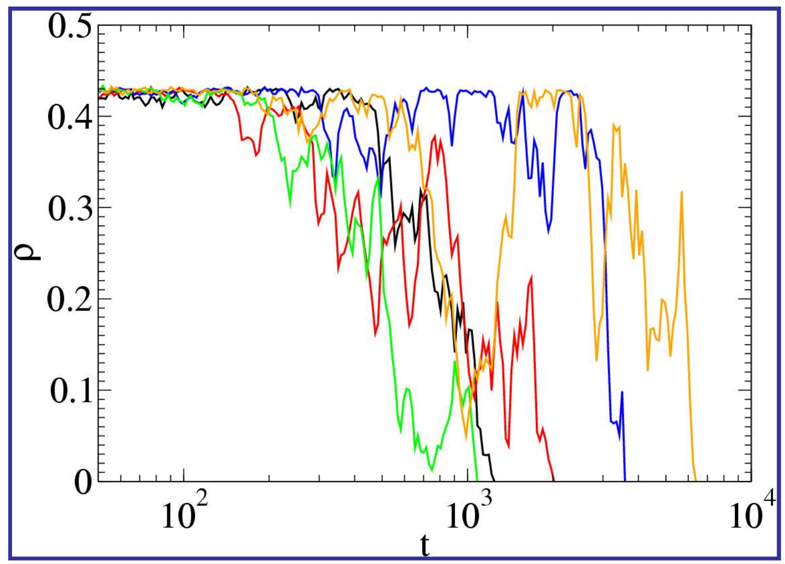
d>2: No Coarsening : Long lived, dynamically active disordered states

Disordered states.



$l = \xi^{-1}$ *Characteristic size of ordered domain*

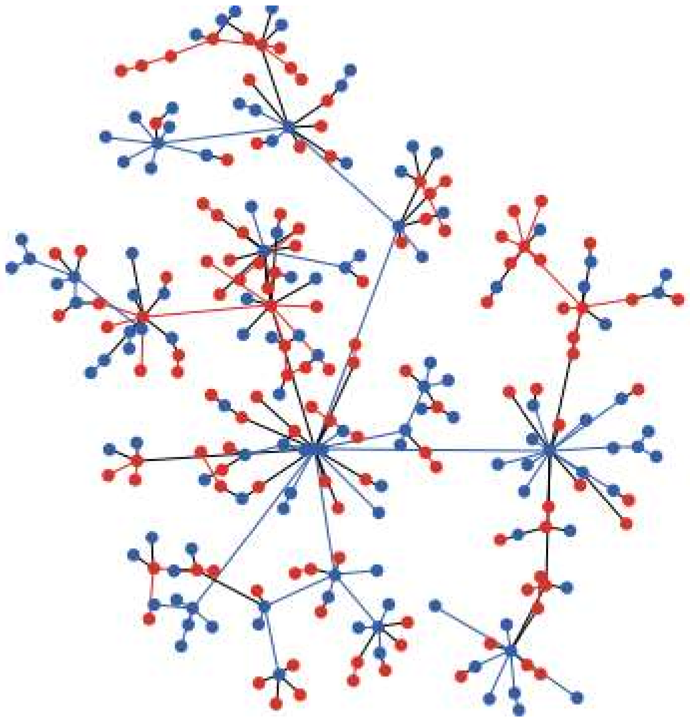
Finite size fluctuations take the system to an absorbing state



$< \rho > \sim e^{-t/\tau}$

Role of topology of interactions

Barabasi-Albert Scale Free Networks

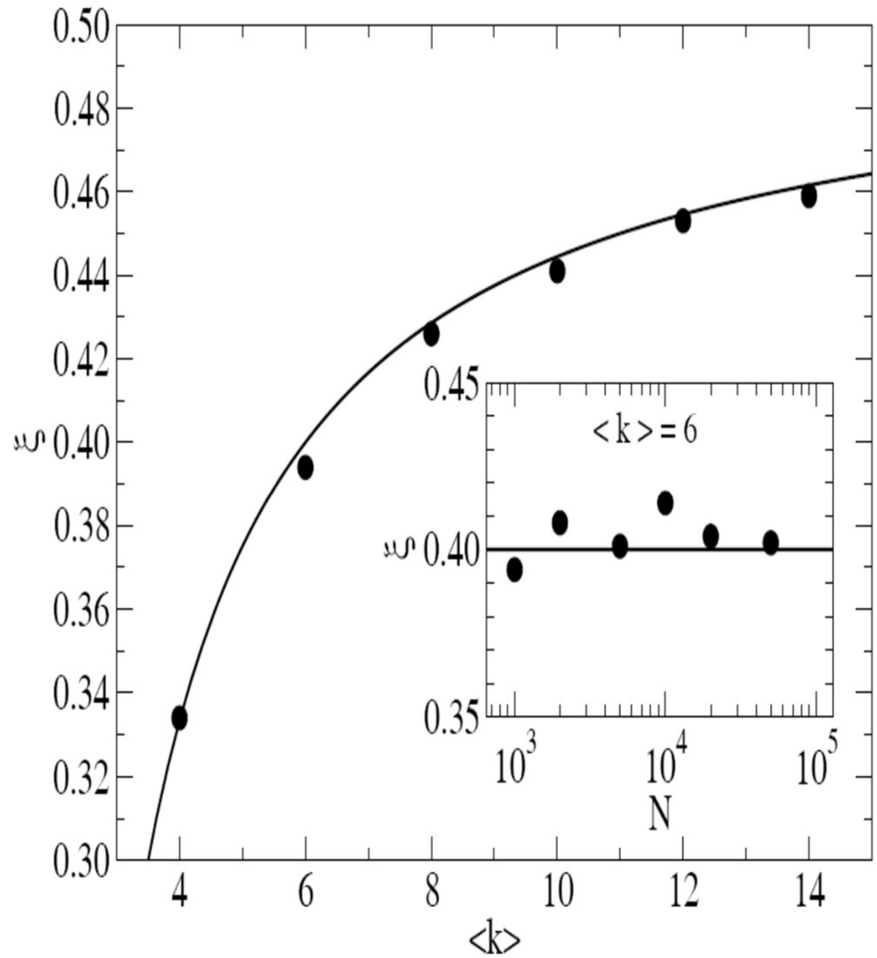


Degree distribution: $P(k) \sim k^{-3}$

Pair approximation for uncorrelated networks

$$\xi = \frac{<k> - 2}{2(<k> - 1)}$$

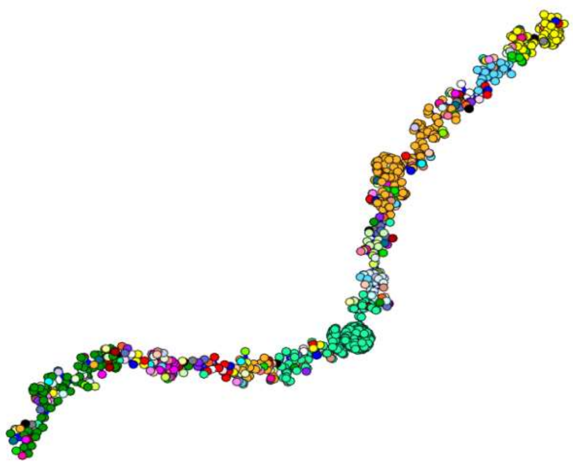
$$l = \xi^{-1}$$ *Characteristic size of ordered domain*



Role of topology?: Dimensionality

1D Scale free net?

*Structured SF: **SSF***

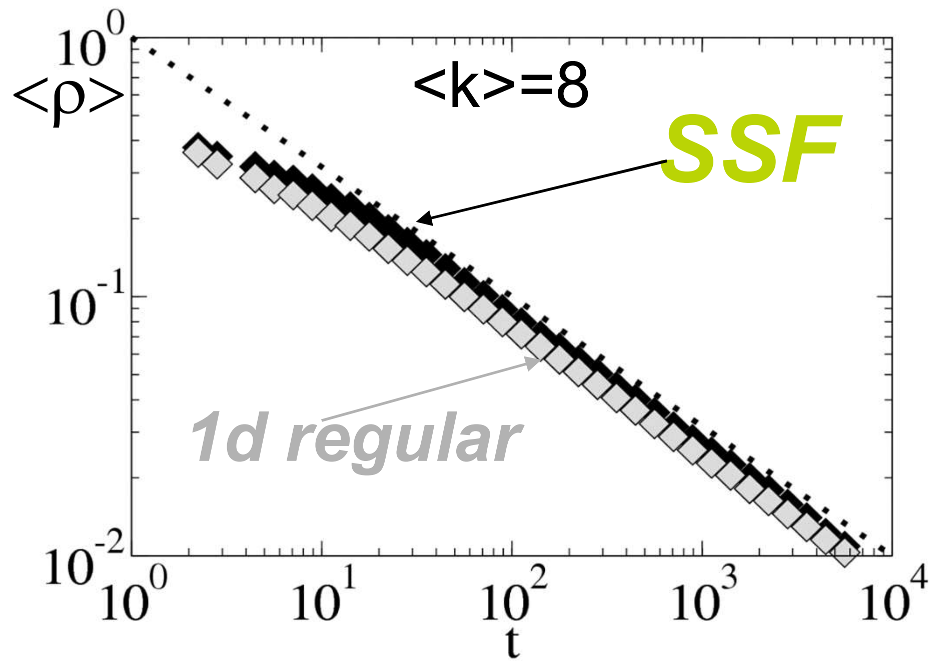


Scale free but
high clustering and 1d

P(k) $\sim$ ***k*** $^{-3}$

L $\sim$ ***N*** ***C*** $\sim$ ***N***^{0}

Klemm and Eguíluz,
*Phys. Rev. E **65**,036123 (2002)*



SSF $<\rho>\sim t^{-1/2}$

$\tau_1 \approx N^2$

***Dimensionality** determines when imitation leads to growing agreement*

Degree** distribution or network **disorder
are not relevant

Coevolving Voter Model *

F. Vázquez, et al, Phys. Rev. Lett. 100, 108702 (2008)

COEVOLUTION:

Dynamics **on** the network coupled with dynamics **of** the network

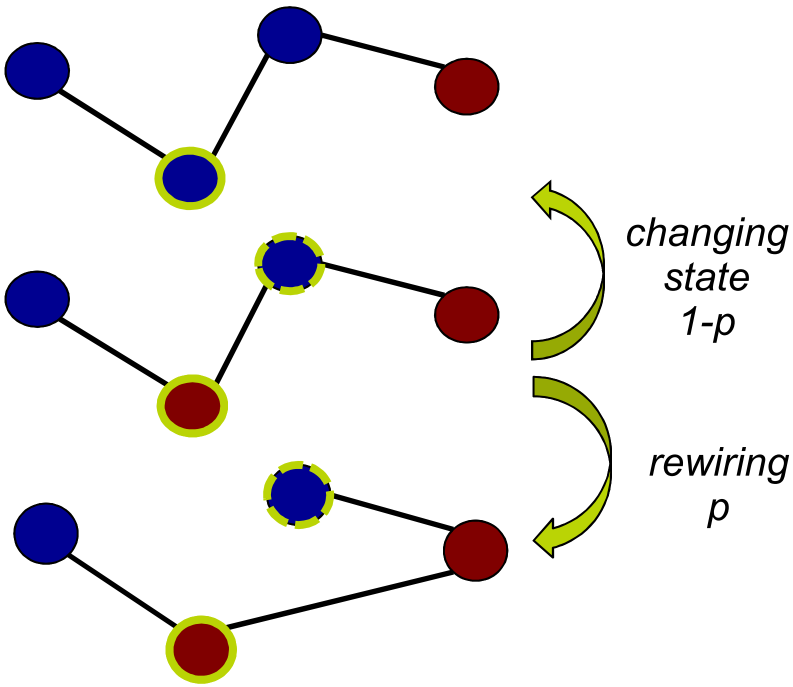
Social Imitation

Voter Model

Breaking and..

..establishing ties

Rewiring



Plasticity p: Imitating vs. choosing neighbors

Coevolving Voter Model

F. Vázquez, et al, Phys. Rev. Lett. 100, 108702 (2008)

Imitation

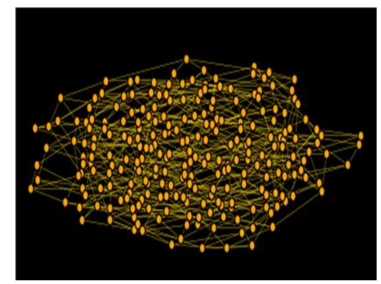
Choosing neighbors

Network Fragmentation Transition

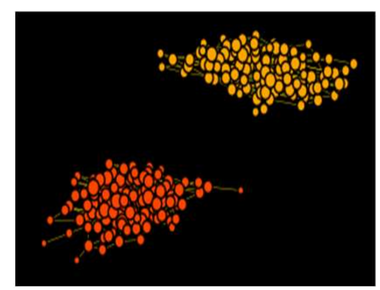
Fragmentation due to **competition** of time scales:

- evolution **of** the network (link dynamics)
- evolution on the network (node state dynamics)

Critical value of plasticity p_c

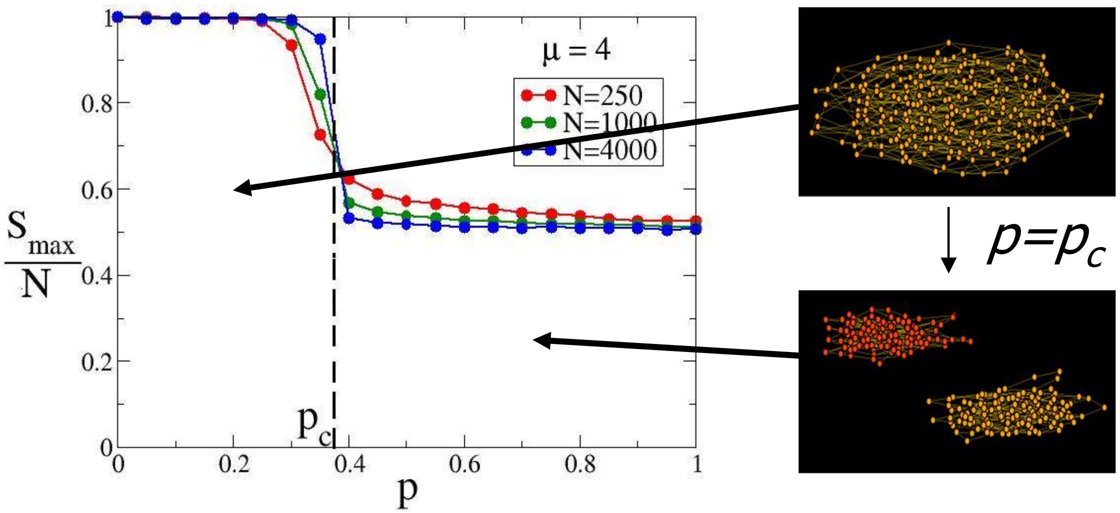


Transition



Fragmentation transition

Size of largest network component.



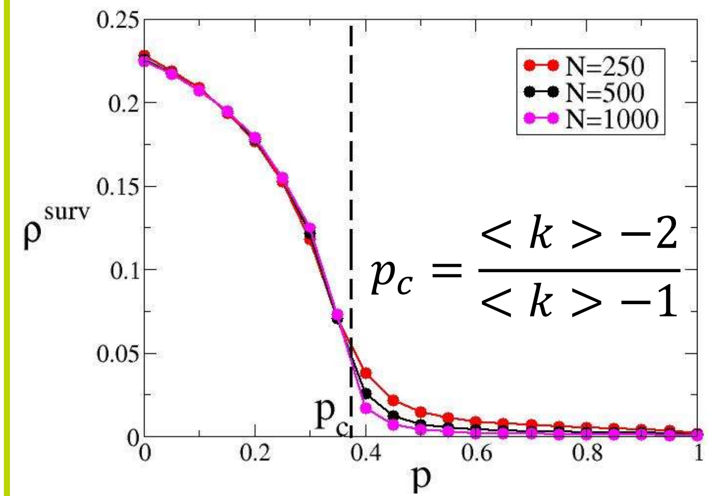
Active phase → Connected network ($S_{max}/N = 1$) ($N = \infty$)

Frozen phase → Fragmented network ($S_{max}/N \approx 0.5$)

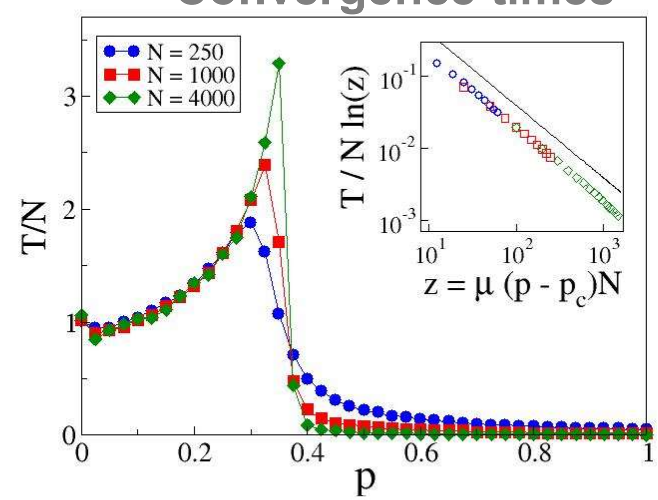
- $p<p_c$: slow rewiring keeps network connected until system fully orders and freezes in a single component.
- $p>p_c$: fast rewiring leads to fragmentation of network into two components before system reaches full order.

Coevolution ⟹ Social Polarization

Active links in surviving runs.



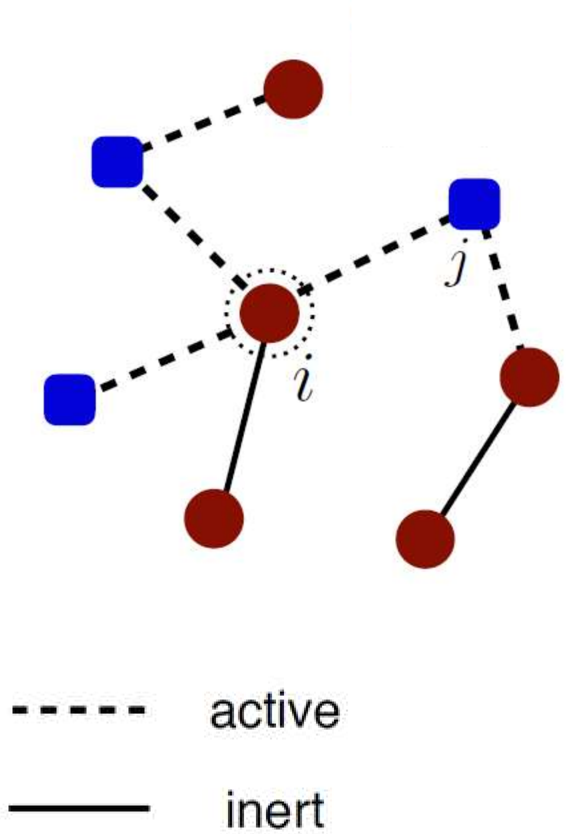
Convergence times



$$T \sim \begin{cases} N\,(p_c - p)^{-1} & \text{for } p \lesssim p_c \\ (p - p_c)^{-1}\,\ln\left[\mu(p - p_c)N\right] & \text{for } p \gtrsim p_c \end{cases}$$

Nonlinear voter model:

Castellano et al PRE (2009); Schweitzer et al EPJB, 2009
Social impact theory, Nowak et al Psychological Rev.1990



Flipping probability of node i: $\left(\frac{a_i}{k_i}\right)^q$

a_i number of active links of node *i*

k_i number of links of node *i*. Degree

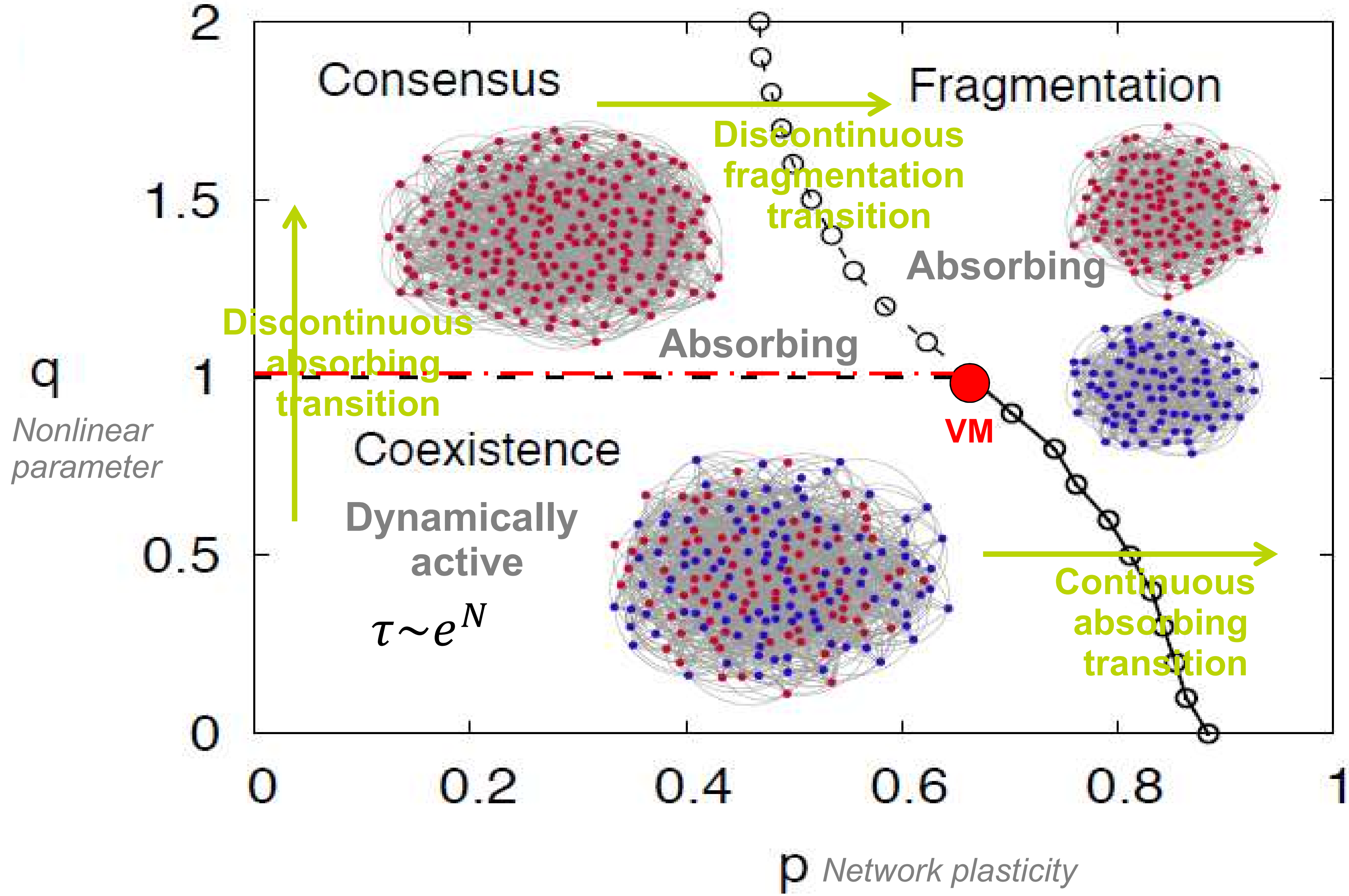
a_i=3
k_i=4

q: Degree of nonlinearity
Nonlinear effect of local majorities

q=1 Voter Model *Neutral situation:*
Random imitation process
q>1 *Probability below random imitation*
q<1 *Probability above random imitation*

Min and San Miguel, Sci. Rep. (2017)

Random Network



Human Activity Patterns: Aging

Fernandez-Gracia et al, Phys. Rev. E (2011); Artime et al, Sci. Rep. 7:7166(2017)

AGING: The longer you remain in a state, less probable to update it

UPDATE RULE:

Each agent is characterized by two variables: state *x* and 'internal time' τ_i

1.with activation probability $p(\tau_i)$ each agent i becomes active. Take $p(\tau) = 1/\tau$

$p(\tau) = 1/N$ corresponds to Monte Carlo Random Asyncrhonous Update

2. active agents update their state x according to voter model dynamical rule

Active agents that change state in step 2 reset $\tau = 0$

3. $\tau_i = \tau_i + 1$

AGING

Activation prob. becomes a function of a *persistence time* τ.

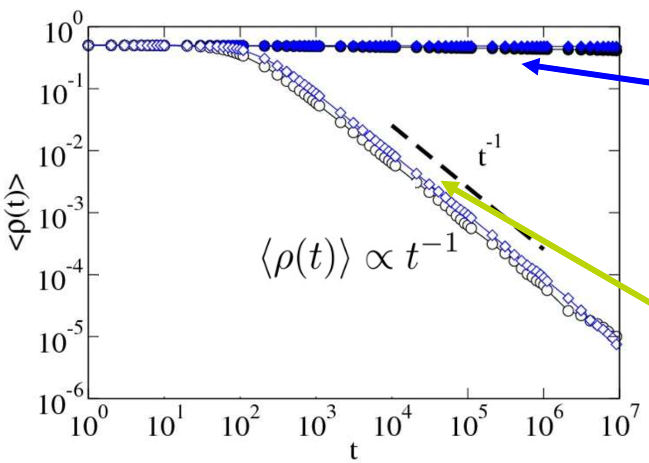
Updating is part of the dynamical model.

Coupled dynamics of state x and 'internal time' τ_i

Fernandez-Gracia et al, Phys. Rev. E (2011); Artime et al, Sci. Rep. 7:7166(2017)

Density of active links

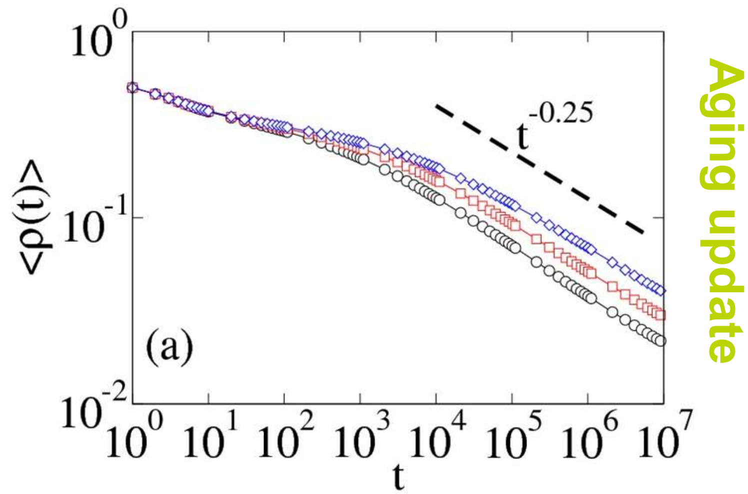
FULLY CONNECTED NETWORK



No aging
No ordering.
Dynamical coexistence

AGING
Ordering

ER RANDOM NETWORK <k>=6

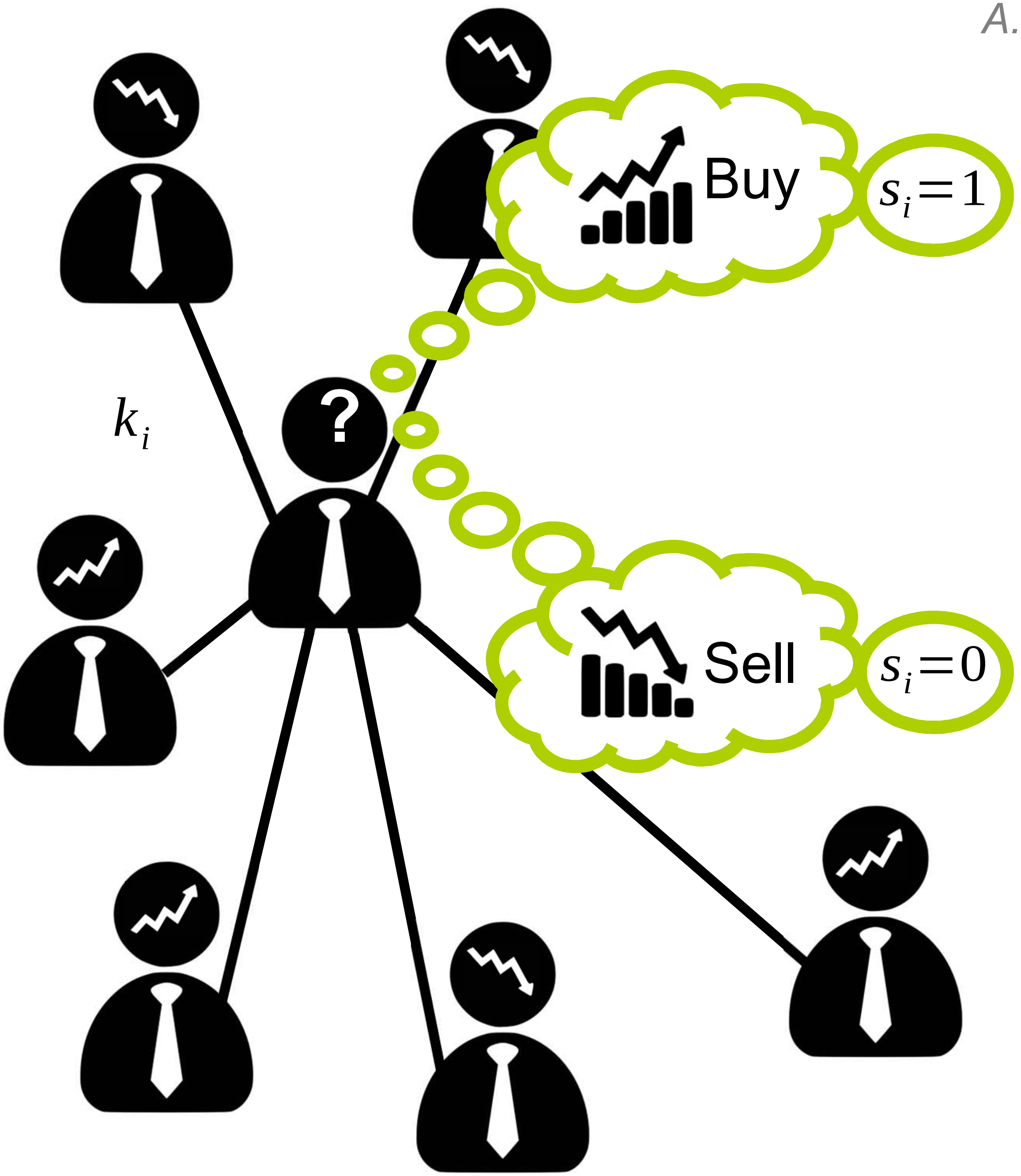


N=1000, 2000, 4000

- Aging induced ordering
- Aging societies more prone to agreement

IMPERFECT IMITATION: The noisy voter model

A. Kirman, Quarterly J. of Economics (1993)



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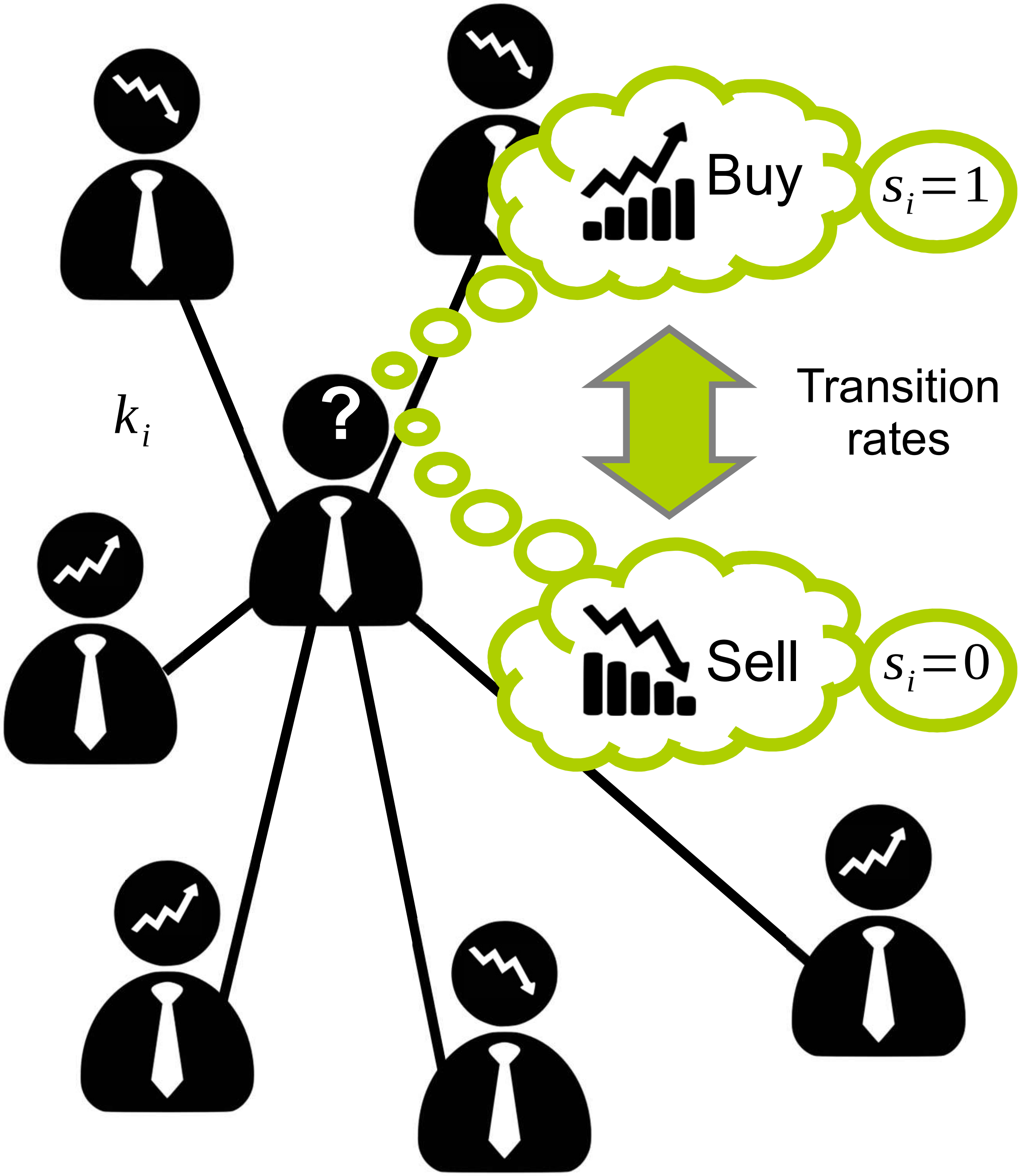
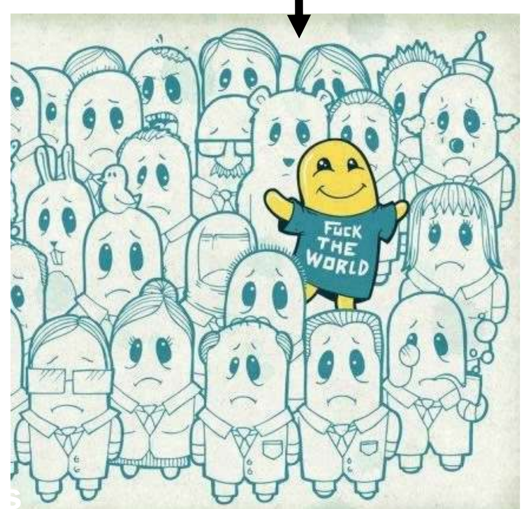


$$r_i^- = a + \frac{h}{k_i} \sum_{j \in nn(i)} (1 - s_j)$$

$$r_i^+ = a + \frac{h}{k_i} \sum_{j \in nn(i)} s_j$$



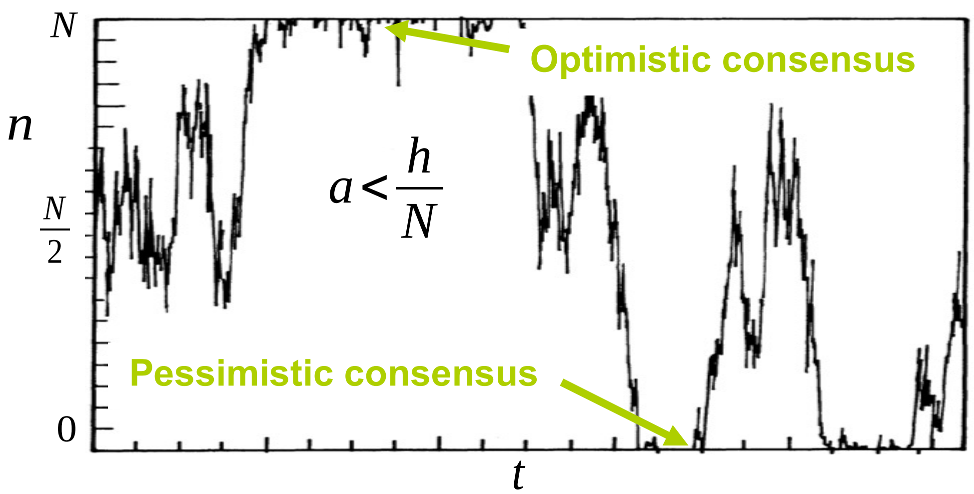
Idiosyncratic

Noise

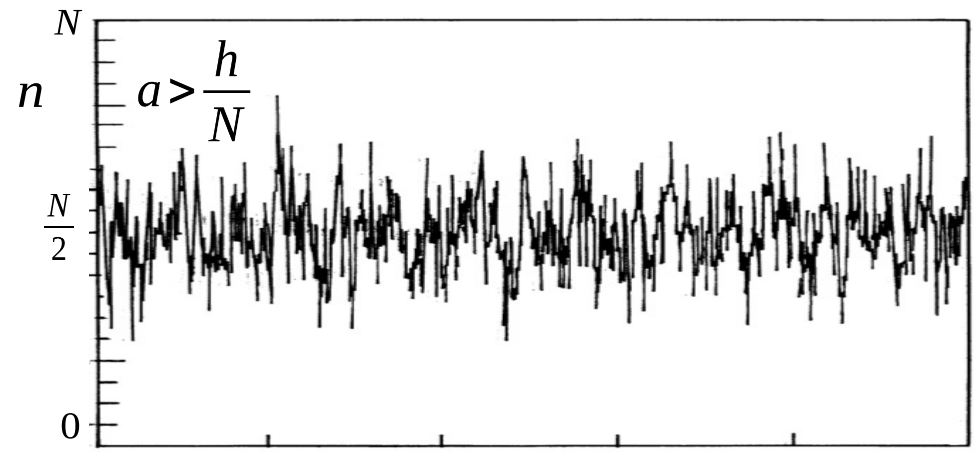
Free will

The noisy voter model: Mean field

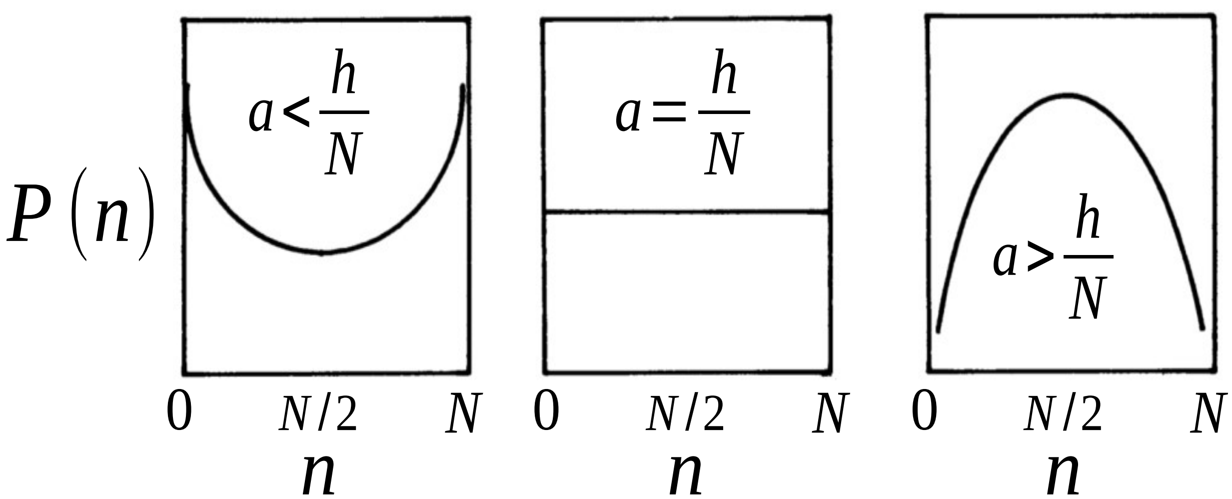
Idiosyncratic behavior < Herding behavior



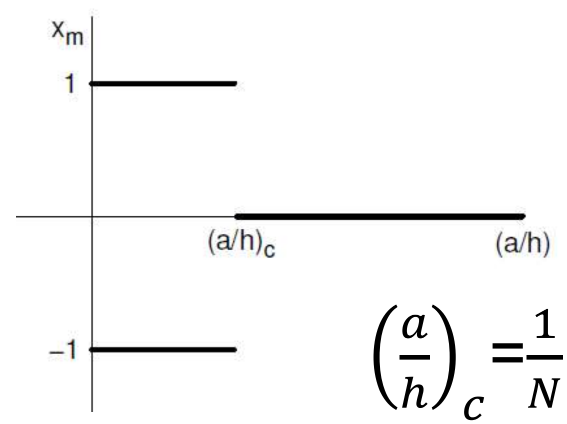
Idiosyncratic behavior > Herding behavior



n = # of agents in state 1



Finite-size discontinuous noise induced transition

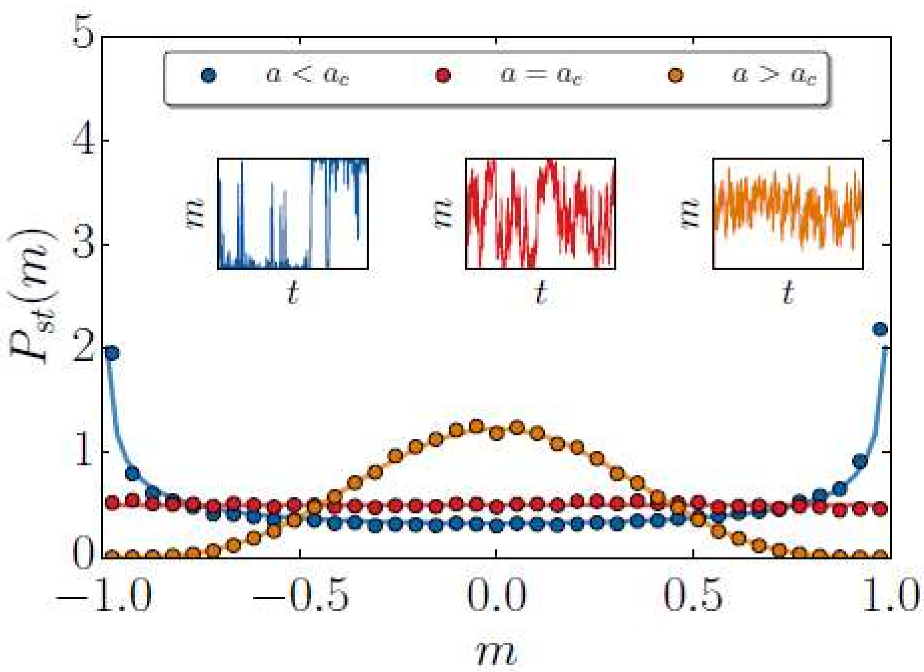


$$\left(\frac{a}{h}\right)_c = \frac{1}{N}$$

Voter Model + Noise + Aging*

Artime et al, Phys. Rev. E **98**, 042143 (2018)

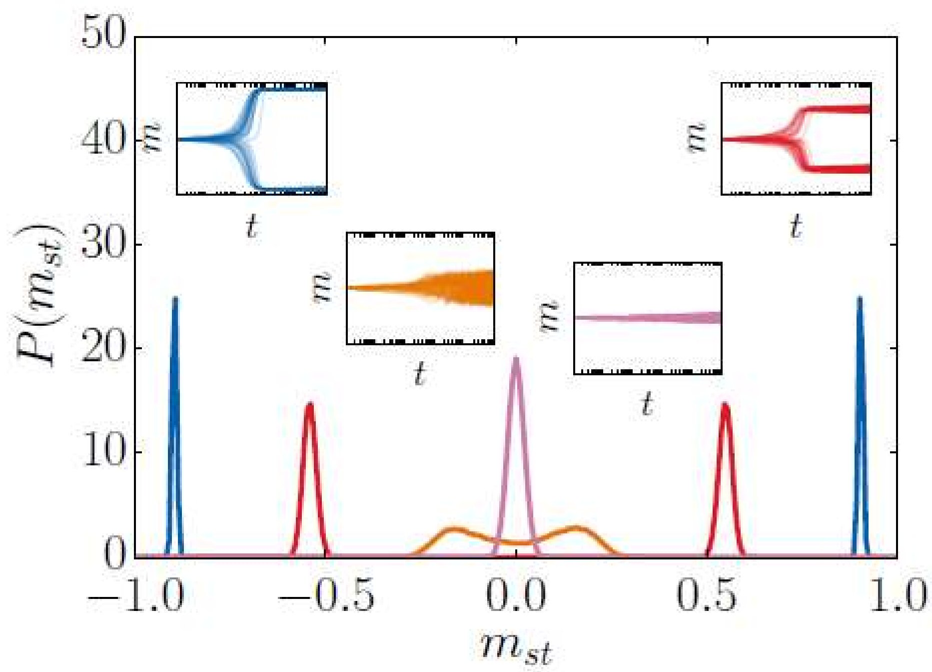
Noisy Voter Model



Noise induced finite size

DISCONTINUOUS TRANSITION

+ AGING

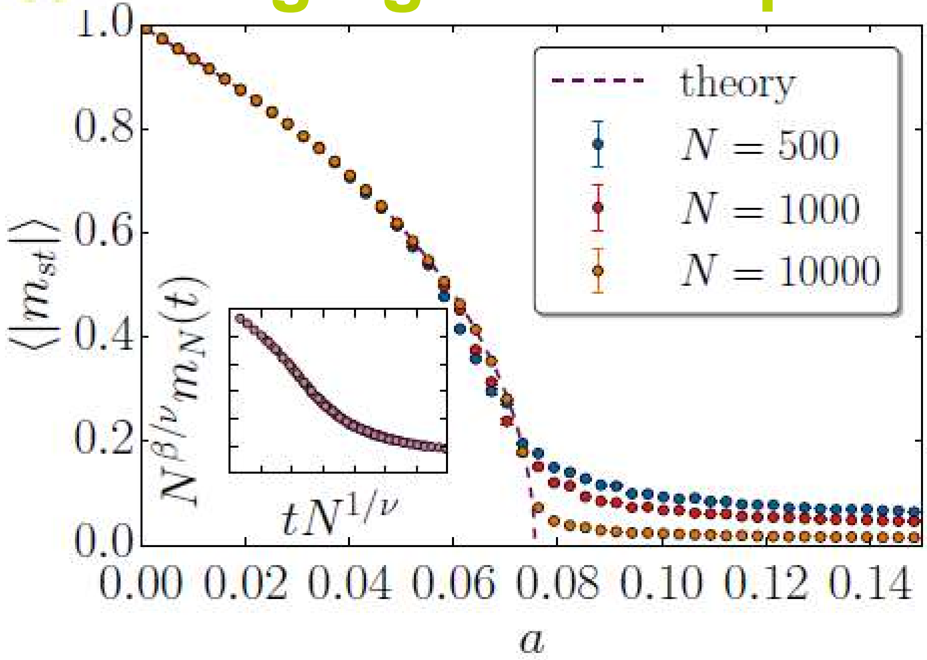


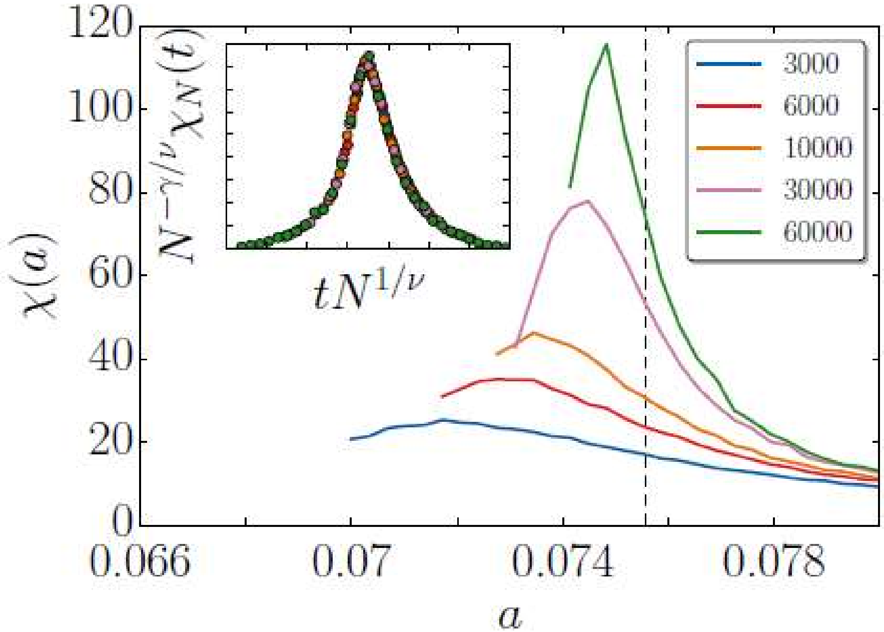
CONTINUOUS

TRANSITION ?

Artime et al, Phys. Rev. E **98**, 042143 (2018)

An aging-induced phase transition



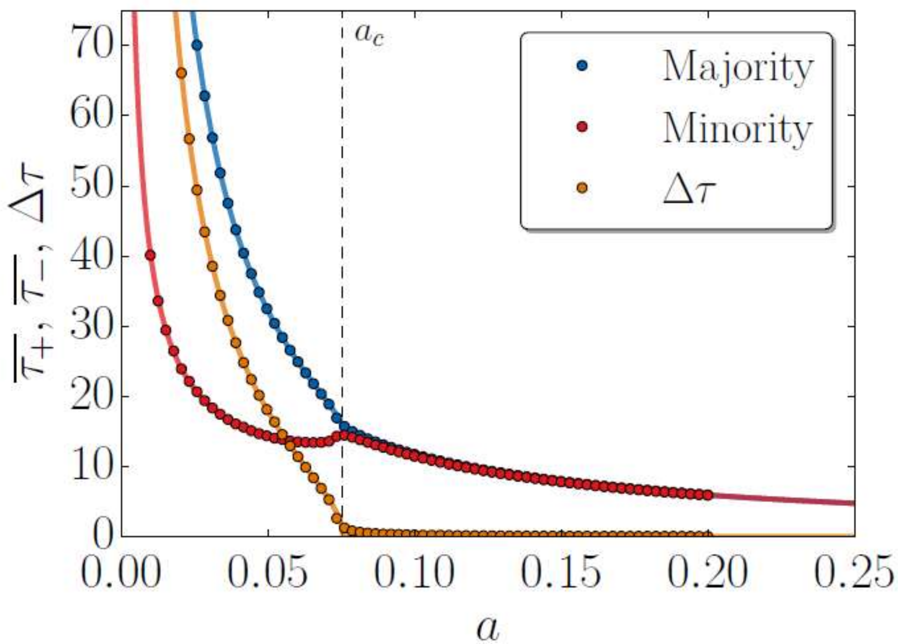


Ising universality class

ER net β=1/2, $\gamma = 1$, $d_c \nu$=2 d_c=4

Ising exponents in d=2,3

Asymmetric aging



Mean internal times

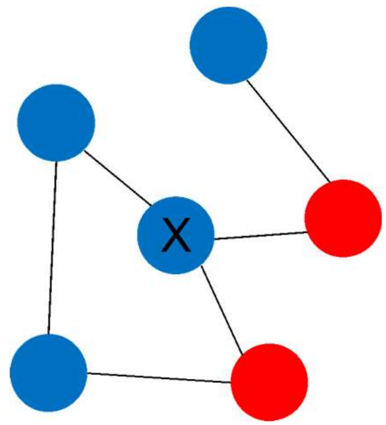
Threshold Model of Complex Contagion

M. Granovetter The American J. Sociol. 83 (1978)

D.J. Watts, PNAS 99, 5766 (2002)

J. P. Gleeson and D. J. Cahalane, Phys. Rev. E 75, 056103 (2007)

Complex Contagion
Social pressure



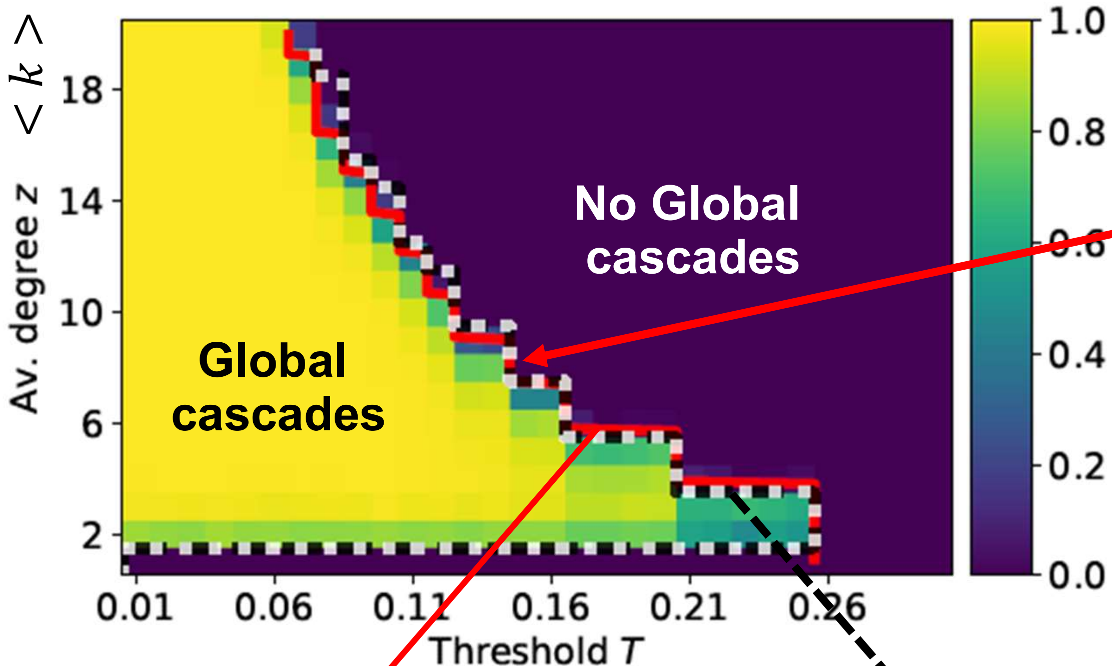
m adopting neighbors

$$P(X \rightarrow X) = \theta\left(\frac{m}{k} - T\right)$$

T: Threshold

k: node degree

Random networks



No aging

Aging

Cascade condition

$$\sum_{k}^{\lfloor 1/T \rfloor} k(k-1)P_k = \langle k \rangle$$

Discontinuous transition for spreading from an initial seed.

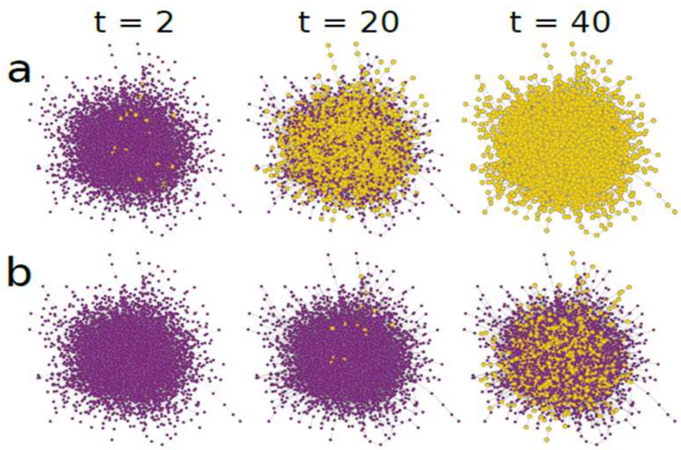
Cascade condition is not modified by aging

Aging in Threshold Model Dynamics

Abella et al., *Phys. Rev. E* **107**, 024101 (2023)

No aging

Aging



Random ER network

N=8000

T=0.2

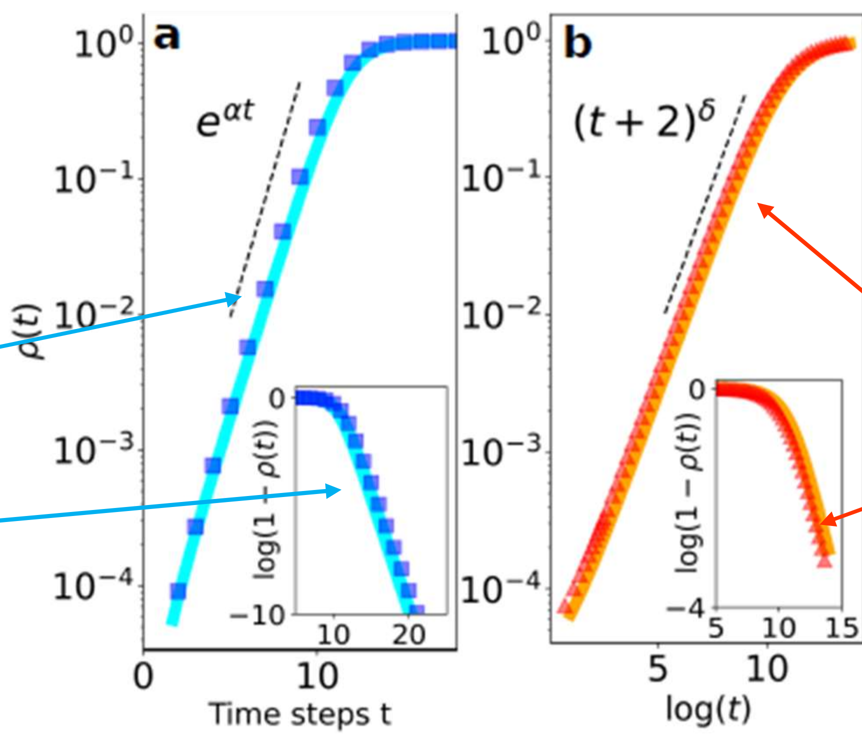
Average degree z=3

Density of adopted agents: From exponentials to power-laws

No aging

$$\rho(t) \sim \rho_0\, e^{\alpha t}$$

$$1 - \rho(t) \sim e^{-\alpha t}$$



Aging

$$\rho(t) \sim \rho_0\, ((t+2)/2)^\delta$$

$$1 - \rho(t) \sim 1/(t+2)$$

— *Numerical integration*

■ *MC Simulation*

Aging in TM: Analytical results

Abella et al., Phys. Rev. E **107**, 024101 (2023)

Markovian dynamics: Rate equations for

$s_{k,m,j}(t)$: density of non-adopted nodes with degree k, m neighboring adopted agents and age j

$$\frac{ds_{k,m,j}}{dt} = -s_{k,m,j} - (k-m)\beta^s s_{k,m,j} + (k-m+1)\beta^s s_{k,m-1,j-1} + F_A(k,m,j-1)\, s_{k,m,j-1},$$

$$\frac{ds_{k,m,0}}{dt} = -s_{k,m,0} - (k-m)\beta^s s_{k,m,0}$$

$$F_A(k,m,j) = 1 - p_A(j)\,\theta(m/k - T)$$

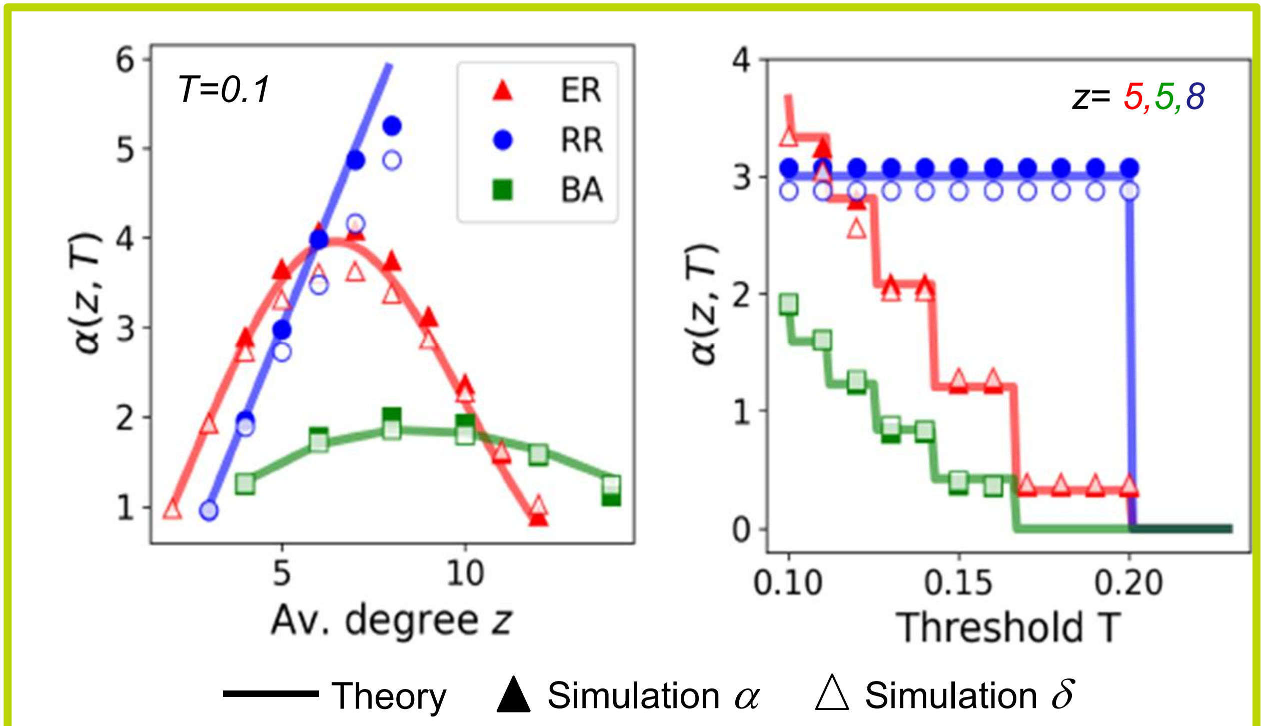
- No aging $p_A(j) = 1$
- **Aging** $p_A(j) = 1/(j+2)$

Density of adopted agents $\rho(t)$

$$\rho(t) = 1 - \sum_j \sum_k p_k \sum_{m=0}^{k} s_{k,m,j}$$

- No aging $\rho(t) \sim \rho_0\, e^{\alpha t}$
- **Aging** $\rho(t) \sim \rho_0\, ((t+2)/2)^{\delta}$

$$\delta(z,T) = \alpha(z,T) = \sum_{k=0}^{\lfloor 1/T \rfloor} \frac{k(k-1)}{z} p_k - 1$$



- Aging results in Heterogeneous Activity Patterns
- Heterogeneous interevent time distributions produce qualitative changes:

Voter Model: From dynamical coexistence of opinions to ordering dynamics

Noisy Voter Model: From finite size discontinuous transition to well defined continuous transition

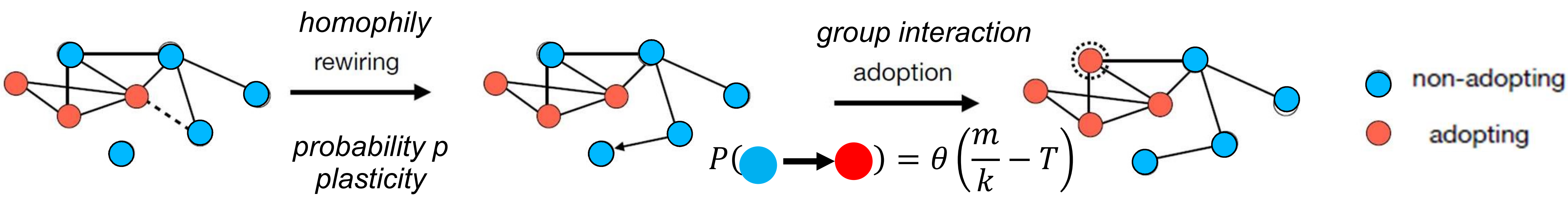
Threshold Model (Complex Contagion): From exponential to power-law cascades

- Beware of social simulations of interacting agents based on a constant activity rate:

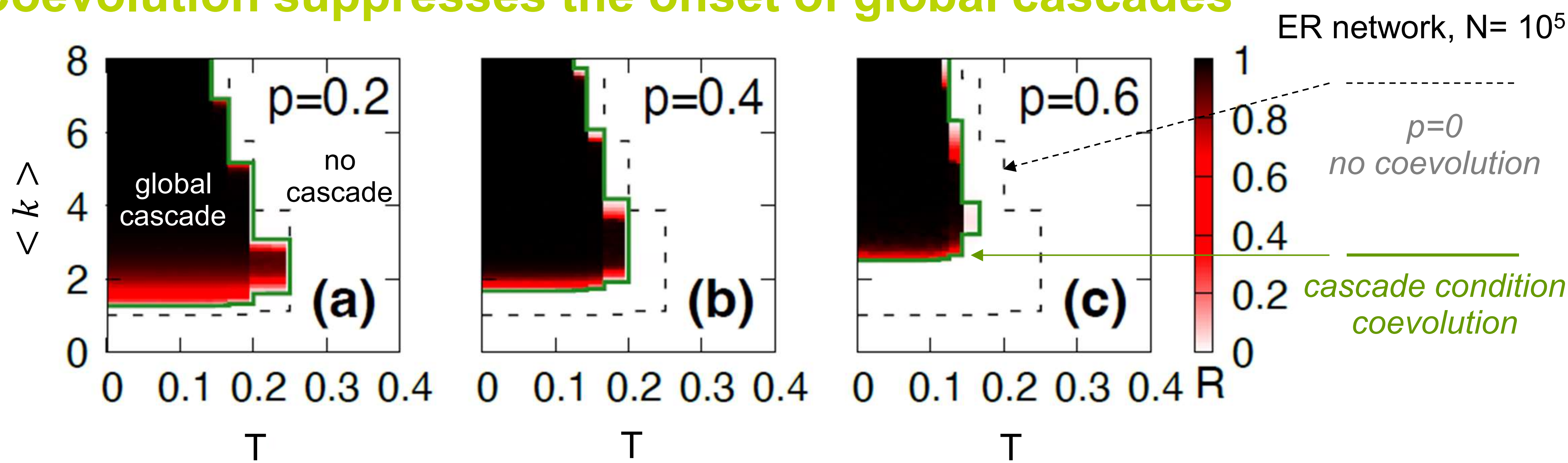
Human activity patterns need to be implemented as an essential part of social simulation.

Co-evolutionary Threshold Model

Byungjoon Min and M. San Miguel, Entropy (2023)

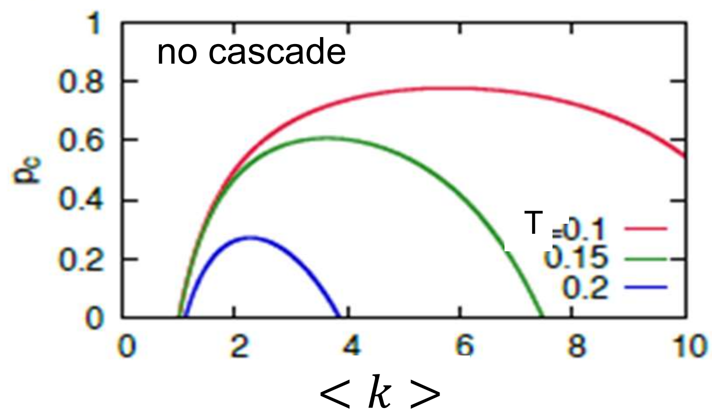


Coevolution suppresses the onset of global cascades



Mean field aprox. Cascade condition

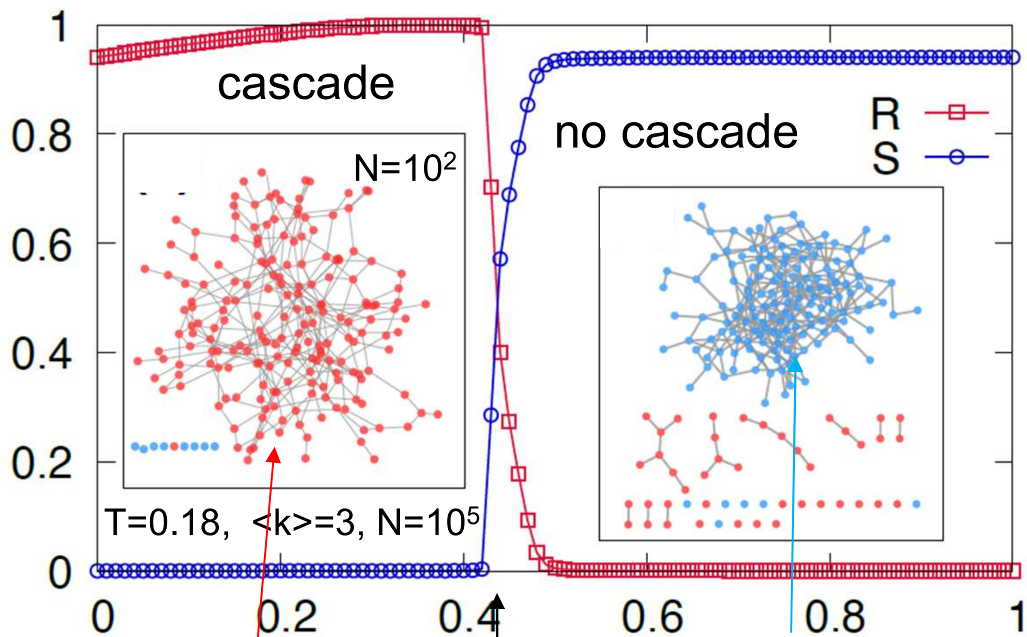
$$(1-p)\sum_{k}^{\lfloor 1/T \rfloor} k(k-1)P_k = \langle k \rangle \xrightarrow{p_c(T, <k>)}$$



Byungjoon Min and M. San Miguel, Entropy (2023)

Mechanism of suppression of global cascades

Segregation of adopting nodes



R — Fraction of adopting nodes

S — Size of largest non-adopting cluster

non-adopted

adopted

Large cluster of adopting nodes

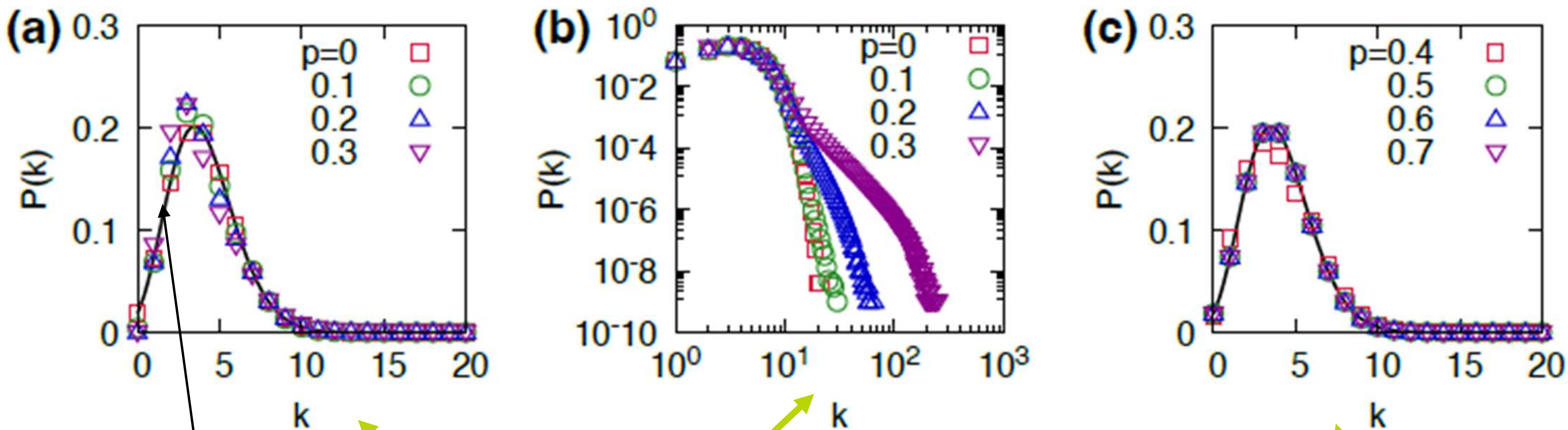
$p_c(T, <k>)$

Large cluster of NON adopting nodes + small adopting clusters

Byungjoon Min and M. San Miguel, Entropy (2023)

Structure of rewired networks

<k>=4, $N=10^5$



Poisson <k>=4

$p<p_c$

Non-poissonian

Cascade regime

$p>p_c$

Poissonian

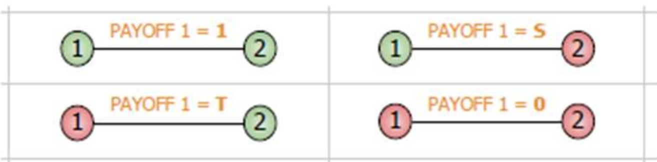
No Cascades

General Coordination Games

Pay-off matrix

Strategy A (green) Strategy B (red)

	2 (A)	2 (B)	
1 (A)	1,1	S,T	$S<0$
1 (B)	T,S	0,0	$T<1$



AA and **BB** are **Nash equilibiria:** No player can improve her payoff by switching to the other strategy

AA equilibrium: pay-off dominant

BB also equilibrium: Although each player is awarded less than optimal payoff, neither player has incentive to change strategy due to a reduction in the immediate payoff

Pay-off (Pareto Dominance) and Risk Dominance

Expected pay-off playing A: $\langle\Pi_A\rangle = \frac{1}{2}\,1 + \frac{1}{2}\,S$

Expected pay-off playing B: $\langle\Pi_B\rangle = \frac{1}{2}\,T + \frac{1}{2}\,0$

$\langle\Pi_A\rangle > \langle\Pi_B\rangle \Rightarrow S+1>T$

- $S+1>T$, **AA risk dominant** equilibrium
- $S+1<T$, **BB risk dominant** equilibrium

Question: Equilibrium selection for *S+1<T.*

AA Pay-off dominant or BB risk dominant ?

EVOLUTIONARY GAME THEORY:

Iteration of two steps for a system of N interacting agents

STEP 1: Each agent plays the game with all her neighbors in a network and accumulates a pay-off

STEP 2: Strategy update by a dynamical rule

REPLICATOR DYNAMICS: Agents choose a neighbor at random: if the payoff of the chosen neighbor is lower than the agents own, nothing happens. If it is larger, the agent will adopt the neighbors strategy with a probability proportional to the difference between the two payoffs.

Alternatives: Unconditional Imitation, Best Response, Moran, Fermi rule…..

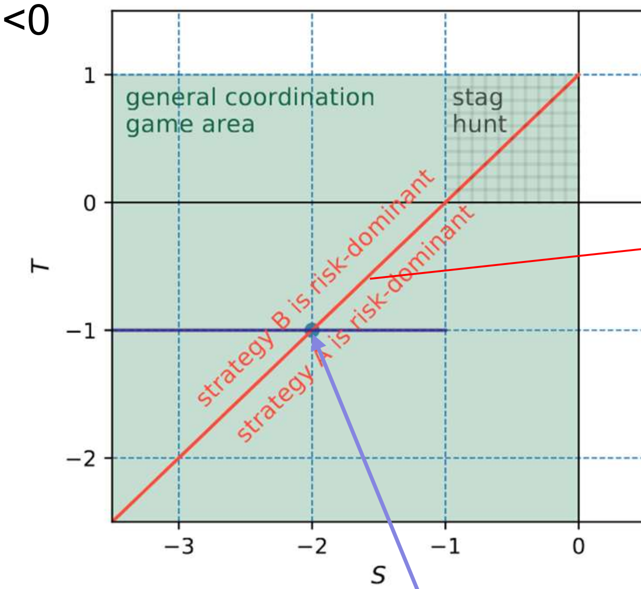
QUESTIONS: i) Coexistence of strategies or consensus?
ii) Equilibrium/Consensus selection?

+ network coevolution

*T. Raducha, M. San Miguel, Sci. Rep. **12,** 3373 (2022)*

General Coordination Game
Equilibrium Selection

T<1, S<0



$$\begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 1 & S \\ T & 0 \end{pmatrix} \end{array}$$

T=S+1

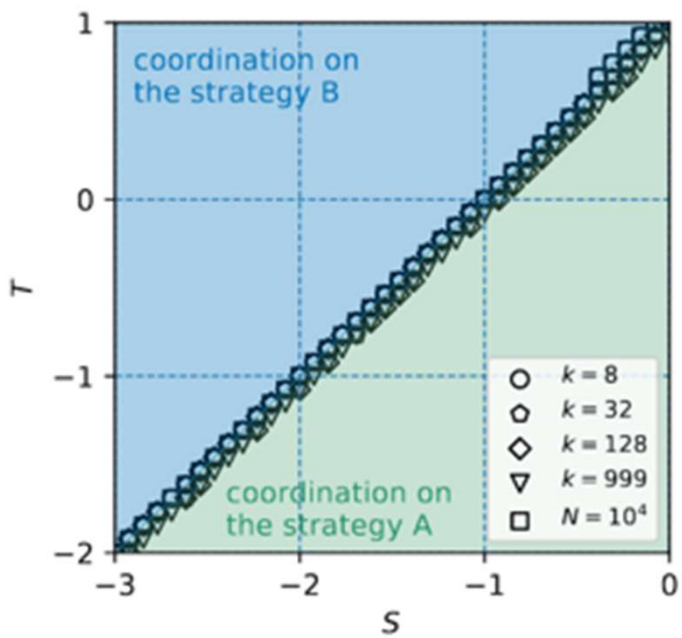
Mean field Replicator Dynamics

$$\frac{\partial \alpha}{\partial t} = (S + T - 1)\alpha^3 + (1 - 2S - T)\alpha^2 + S\alpha.$$

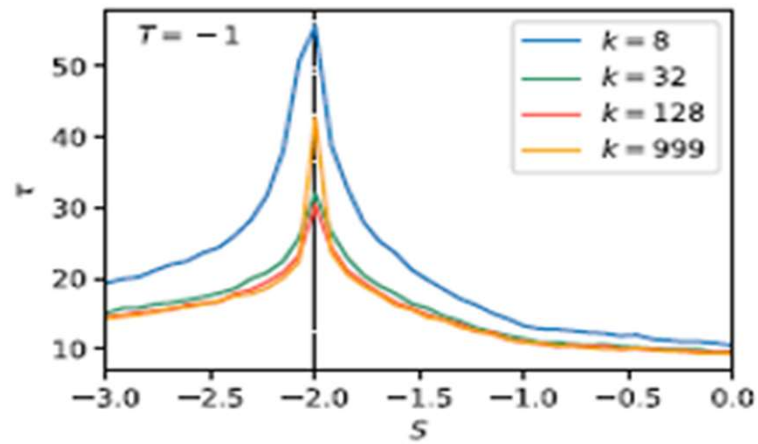
α proportion of agents playing A

Risk-dominant equilibrium selected

Random network



Time to coordination



Risk-dominant equilibrium selected
Mean field transition line valid for any <k>

COEVOLUTION IN COORDINATION GAMES

Gonzalez Casado et al , Sci. Rep. ***13,****2866(2023)*

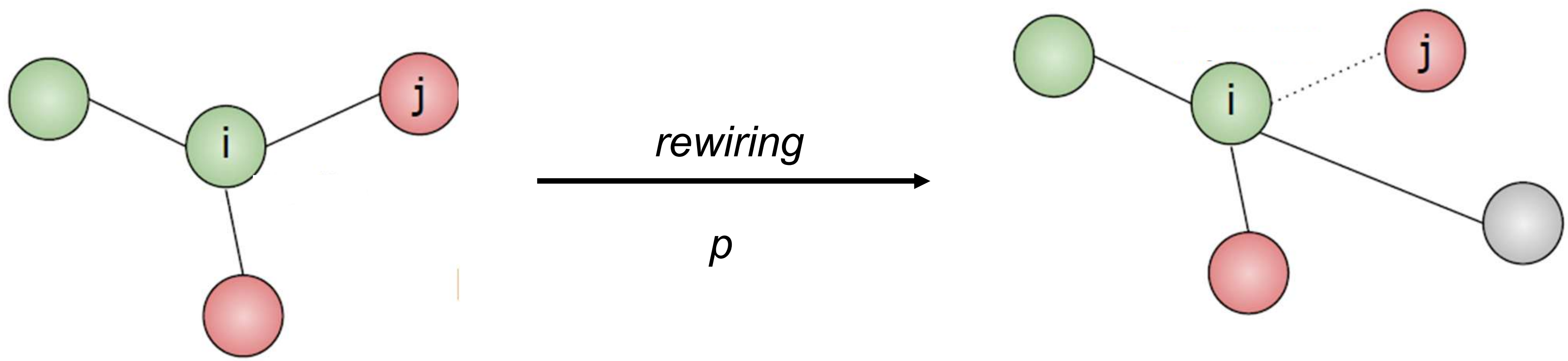
Two strategies A B

STEP 1: Each agent plays the game with all her neighbors in a network and accumulates a pay-off

STEP 2: Strategy update and **network evolution**

Select randomly agent i and neighbor j. If strategy of i and j are different:

i) With probability *p* random rewiring



ii) With probability *1-p* use the evolutionary dynamics update rule (Replicator Dynamics)

Question: Coordination or Fragmentation ?
Equilibrium selection?

General Coordination Game

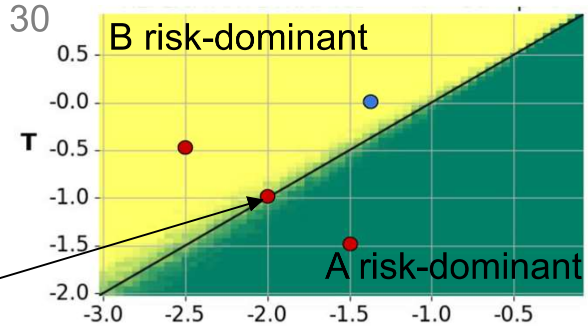
ER network, <k>= 30
N=1000

$$\begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} A \quad\; B \\ \begin{pmatrix} 1 & S \\ T & 0 \end{pmatrix} \end{array}$$

A always Pay-off dominat

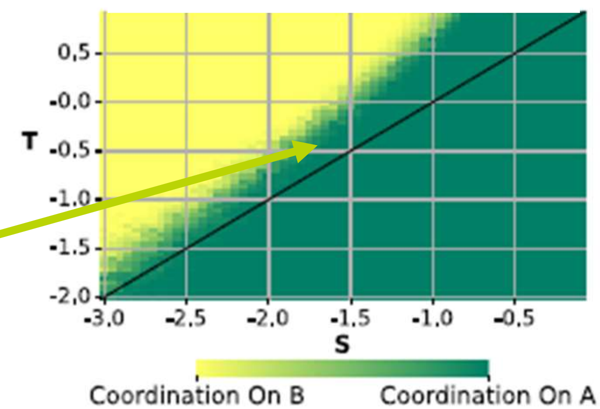
α = 1: all players choose A

α = 0: all players choose B

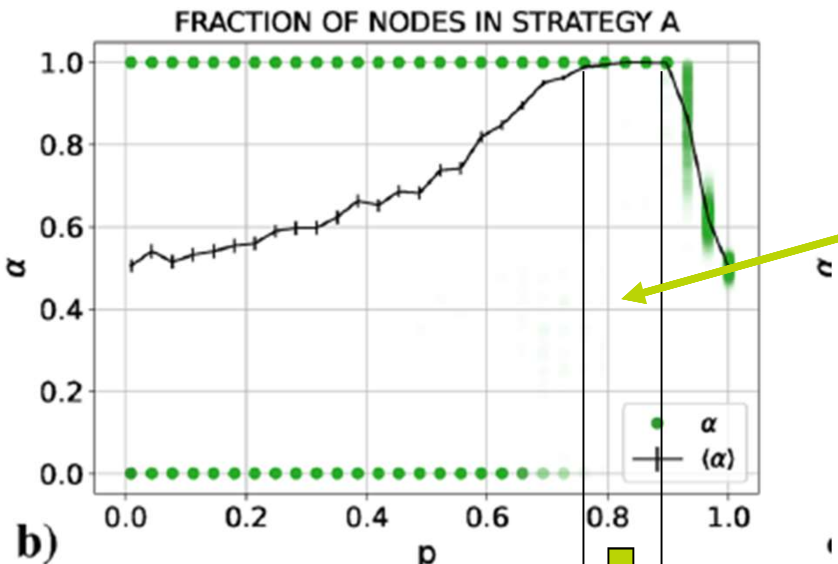


p=0

Close to transition line S+1=T *(p=0)*

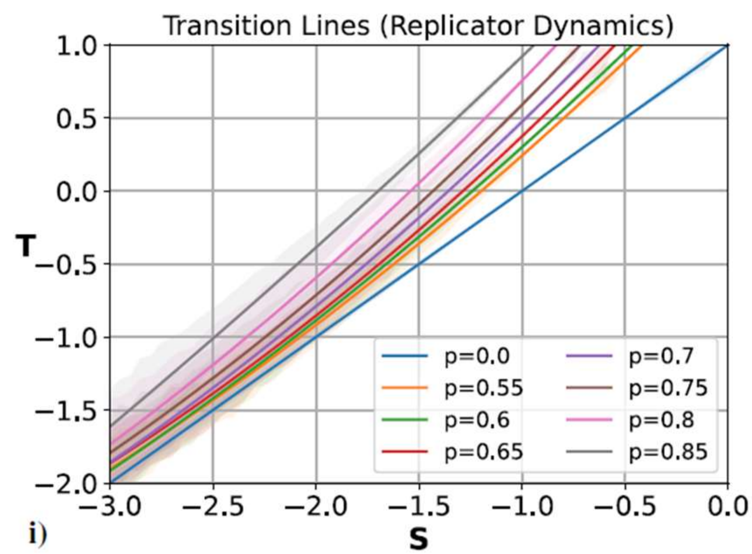


p=0.8



New transition line p_c p_f Fragmentation

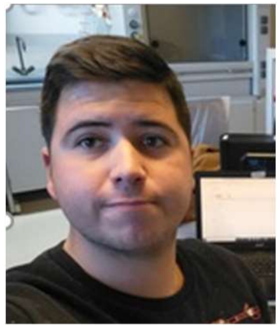
Coordination in A



Coevolution creates a region for global coordination in pay-off dominance

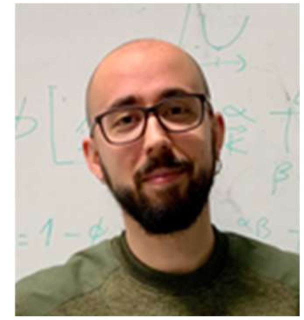
THANKS!

Miguel Angel Gzlez-Casado



David Abella

Toni Fdez-Peralta



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Raúl Toral