Linear Response Theory in hard colloids

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In mirorheology, a colloidal tracer is inserted in a soft matter system and its dynamics is monitored. This tracer bead can be subjected to an external force, in so-called active microrheology. In passive microrheology, on the other hand, the tracer moves in response to the bath equilibrium thermal or density fluctuations. In active microrheology, the effective friction coefficient shows three regimes as a function of the external force: a linear regime at low forces, a non-linear force thinning regime, and a high force regime, dominated by the ballistic collisions with bath particles, that can be linear or non-linear. The connection between passive microrheology and (the linear low-force regime of) active microrheology is provided by linear response theory, which provides both the transient and stationary response of the tracer particle to the external force based on the equilibrium fluctuations:

$$\langle z(t)\rangle \;=\; \frac{\beta F_{ext}}{6}\langle \delta r^2(t)\rangle_{eq} \tag{1}$$

where $z(t)$ is the tracer displacement when force F_{ext} is applied, and $\langle \delta r^2(t)\rangle_{eq}$ is the equilibrium mean squared displacement.

In this work, simulations of passive and active microrheology in a system of quasi-hard spheres will be presented. The tracer is pulled with a constant force, and its radius is varied from a to $8a$, where a is the radius of the bath particles. The tracer diffusion coefficient (in passive microrheology) and friction coefficient at low forces (in active microrheology) fulfill the Stokes-Einstein relation, and the tracer velocity autocorrelation function (VACF) correctly predicts the transient tracer velocity upon application of the external force. Both results, and further tests, confirm the validity of the linear response theory in this system. The friction coefficient and inverse diffusion coefficient are also interpreted using the Brinkman model, although our interpretation of the parameters is different from the original formulation. Finally, the effects of the moving tracer on the bath are presented.

A model based on the mode coupling description of the bath and tracer particles (as a binary mixture) is also presented to comprehend the physical mechanisms in passive microrheology. The theory requires the tracer-bath cross structure factor, which is calculated within the Percus-Yevick approximation for mixtures, with excellent agreement with the simulations, and the model correctly predicts frequency dependence of the tracer VACF, which implies the transient tracer response, as mentioned above.