Amplitude modulation control of spatiotemporal chaos in starlike networks of damped-driven pendula

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Introduction

Applying amplitude modulations to a parametrically excited damped pendulum is shown to be a reliable method to control (suppress or enhance) its chaotic behaviour. Analytical (Melnikov analysis) and numerical (Lyapunov exponents and bifurcation diagrams) results show an effective control scenario. The method's effectiveness at suppressing spatiotemporal chaos of starlike networks of sinusoidally coupled chaotic pendula is demonstrated where effective regularization is obtained under localized control on an increasing number of pendula.

A single pendulum case

We consider the equation of a dissipative pendulum whose pivot is subjected to a vertical oscillation having a small resonant amplitude modulation:

$$\ddot{\theta}+\sin\theta=-\delta\dot{\theta}-\gamma\left[1+\varepsilon\cos\left(\Omega t+\phi\right)\right]\cos\left(\omega t\right)\sin\theta, \quad (1)$$

where $-\gamma\varepsilon\cos\left(\Omega t+\phi\right)\cos\left(\omega t\right)\sin\theta$, is the control excitation. When the control excitation is absent ($\varepsilon=0$), we assume that the pendulum has a chaotic attractor for a given set of the remaining parameters. A summary of some of the results is shown in Fig. 1.

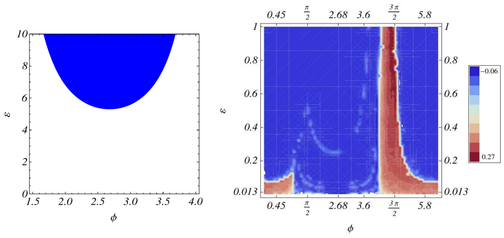

Fig. 1. Typical boundary function encircling the regularization regions where homoclinic bifurcations are frustrated obtained from Melnikov analysis (left) and maximal Lyapunov exponent obtained numerically (right) in the $\phi-\varepsilon$ parameter plane. The quantities plotted are dimensionless.

Controlling chaos in startlike networks

We will study the application of the above control scenario to a topology consisting of a starlike network (one hub and $N-1$ peripheral leaves) where each node is occupied by a chaotic pendulum and where the CC excitation is solely applied to a number, M, of pendula:

$$\begin{aligned}\ddot{\theta}_i+\sin\theta_i &= -\gamma\left[1+\sigma_H F_c\left(t\right)\right]\cos\left(\omega t\right)\sin\theta_i \\ &\quad -\delta\dot{\theta}_i+\lambda\sin\left(\theta_H-\theta_i\right), \\ \ddot{\theta}_H+\sin\theta_H &= -\delta\dot{\theta}_H-\gamma\left[1+\sigma_i F_c\left(t\right)\right]\cos\left(\omega t\right)\sin\theta_H \\ &\quad +\lambda\sum_{i=1}^{N-1}\sin\left(\theta_i-\theta_H\right), \quad (2)\end{aligned}$$

$i=1,\ldots,N-1$. These equations describe the dynamics of a highly connected node (or hub), θ_H, and $N-1$ linked pendula (or leaves), θ_i, with $F_c(t)\equiv\varepsilon\cos\left(\Omega t+\varphi\right)$ being the (local) CC excitation, while σ_H (σ_i) is equal to 1 when the CC excitation acts on the hub (leaf i) and 0 otherwise, while λ is the coupling constant. Main results are shown in Figs. 2 and 3.

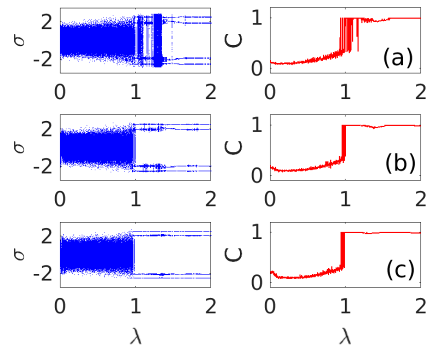

Fig. 2. Typical bifurcation diagrams of the average velocity σ and correlation function C as functions of the coupling parameter λ ($N=10$, and the hub being the node 10). (a) $M=1$. (b) $M=2$. (c) $M=3$. The quantities plotted are dimensionless and all pendula are initially desynchronized.

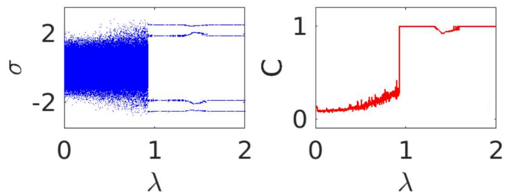

Fig. 3. Typical bifurcation diagrams of the average velocity σ and correlation function C as functions of the coupling parameter λ for $N=10$. All pendula are asynchronous at $t=0$. The hub (node 10) is the single pendulum subjected to multiplicative control. The quantities plotted are dimensionless.

Conclusions

We have shown theoretically and numerically that the application of suitable amplitude modulations is a reliable procedure to control (suppress and enhance) the chaotic behaviour of both isolated parametrically excited damped pendula and starlike networks of them subjected to sinusoidal coupling.

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