Evolutionary Kuramoto Dilemma: Coevolution of Synchronization and Cooperation in Costly Networked Interactions

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The phenomenon of synchronization in coupled oscillating systems has been a subject of substantial interest, given its pervasive nature across a variety of fields [1]. From the rhythmic flashing of fireflies, circadian rhythms, to the periodic oscillations in mechanical, electrical, and quantum systems, synchronization finds diverse applications [2]. The patterns of interaction among individual oscillators are commonly modeled as a network or a graph, significantly impacting the genesis of the synchronized state [3]. Over the years, significant research efforts have delved into understanding the evolution of synchronization in populations of oscillators configured within a network [4]. These networks, representing the intricate web of interactions, are integral to dissecting the complexities of these dynamical systems.

However, many studies assume that the transition of an oscillator's state, necessary for achieving synchronization, is costless. While this assumption is convenient for theoretical modeling, it seems somewhat unrealistic. A more practical hypothesis would involve acknowledging that the modification of an oscillator's state involves a certain cost. This hypothesis brings forth a dichotomous scenario: an oscillator may decide to bear the cost necessary to alter its state, aligning it with the others, or it may remain in its current state, anticipating that the other oscillators will adjust their rhythm [5]. Interpreting this from a game-theoretic perspective, the former choice mirrors an act of cooperation, while the latter represents a choice of defection [6].

Complex networks play a vital role in the emergence of cooperative behavior, especially the presence of highly connected nodes, or hubs, in scale-free networks. Given this, it becomes essential to explore the mechanisms responsible for the onset of synchronization in a network of oscillators where each node must choose whether to cooperate by synchronizing their states with those of their neighbors or not. This line of inquiry leads us towards a coevolutionary approach, intertwining the dynamics of synchronization with game theory [7].

In this talk, I will present a novel coevolutionary model, which is built upon the combination of Kuramoto oscillators playing an evolutionary game [8]. We delve into the emergence of cooperation and synchronization in three different network topologies: Erdös-Rényi random graphs (ER), Random Geometric Graphs (RGG), and Barabsi-Albert scale-free networks (BA). This comprehensive analysis provides a detailed view of the coevolutionary dynamics, revealing the principles that govern the behavior of these fascinating oscillating systems [9].

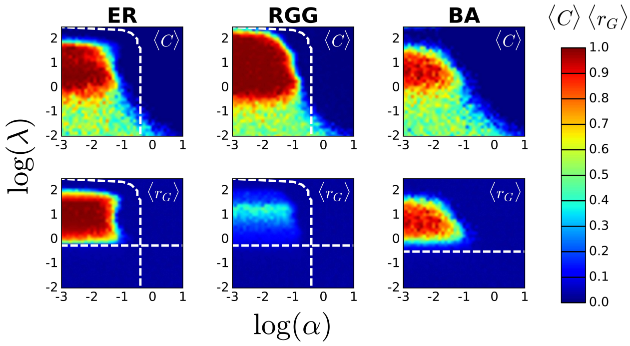

Fig. 1. Emergence of cooperation and synchronization at global scale. The top (bottom) row illustrates the average level of cooperation (synchronization) $\langle C \rangle$ ($\langle r_G \rangle$) as a function of the coupling λ and relative cost α. Each column corresponds to a different topology, namely, ER, RGG, and BA. Results are averages over 50 different realizations.

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