Feedback flashing ratchet: Markovian description, H-theorem and second principle

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Flashing ratches are systems where a directed motion of a Brownian particle could be created just by switching on/off a periodic potential. The key is the alternation between two processes: the motion under the action of the potential (while it is on) and free diffusion (while it is off). The protocol for switching the potential can be (i) an open-loop protocol (periodic, random, etc.), where no information of the system is used in the switching, and (ii) a feedback protocol (or closed-loop protocol), for which the switching is based on the information extracted from the system. In the latter case, the directed movement may emerge even for symmetric potentials.

The system that we have focused on is a feedback flashing ratchet, in which the external control measures the position $x(t)$ of an overdamped Brownian particle at regular times t_k [1]. A schematic representation of the system is depicted in Fig. 1. Depending on the information extracted, i.e. the value of $x(t_k)$, the control takes the value $C = 1$ or $C = 0$ and a sawtooth periodic potential $V(x)$ is thus switched on/off in the time interval $I_k = (t_k, t_{k+1})$, i.e. the particle feels a force $-CV'(x)$ during that time interval.

The process of measurement entails an entropy reduction of the particle, since the information extracted by the control (and employed in the update of the potential) concentrates the probability distribution of the particle position in the microstates compatible with the result of the measurement. Very recently, it has been shown that this entropy reduction (i) can be accurately computed by measuring the entropy of long-enough sequences of control actions and (ii) it is essential to define a physically meaningful efficiency of the ratchet [2].

In this work, we present an alternative framework for the study of the feedback flashing ratchet. In particular, we show that (x, C), i.e. the position of the particle together with the state of the control, is a Markovian stochastic process for which the joint probabilty $P(x, C, t)$ obeys a differential Chapman-Kolmogorov equation [3]. Specifically, one has

$$\partial_t P = \mathcal{L}_{FP}^{(C)} P + \sum_k \delta(t - t_k)[-\Theta_{1-C}(x) P(x, C, t) + \Theta_C(x) P(x, 1 - C, t)] \,. \tag{1}$$

where $\mathcal{L}_{FP}^{(C)}$ is the Fokker-Planck operator for the potential $CV(x)$, and $\Theta_C(x) = 1$ ($\Theta_C(x) = 0$) where x would make the control be switched to the value C. The second term on the rhs of Eq. (1), which only acts at the measurement times t_k, accounts from the transitions from $(x, 1)$ to $(x, 0)$ (or vice versa) when the control is switched off (on) at the times t_k.

The Markovian character of the (x, C) process can be intuitively understood, as shown in Fig. 1. At $t = t_k$, when a measurement takes place, the control value is updated and its value for $t = t_k^+$ depends on the particle position just before the measurement, $x(t_k^-)$. Afterwards, the control value remains constant during the time interval $I_k = (t_k, t_{k+1})$, i.e. until the next measurement. In that time interval I_k, $x(t)$ evolves following an overdamped Langevin equation with potential $CV(x)$. This structure of measurement and Langevin evolution is periodically repeated, showing that the vector (x, C) at time t^+ just depends on its previous value at time t^-.

In this work, we analyse in depth some physical consequences arising from this Markovian description. In particular, we show that it is possible to derive an H-theorem for the differential Chapman-Kolmogorov equation, supporting the existence of a long-time regime in which the ratchet reaches a time-periodic state. We also discuss the implications of this result for the thermodynamic balance, in order to improve our understading of the second principle—with the contribution of the information gathered by the control.

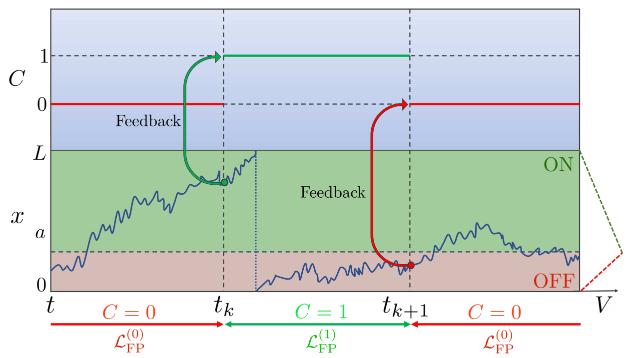

Fig. 1. Schematic evolution of the process (x, C). In the considered interval, two measurements of the control at times t_k and t_{k+1} take place. By itself, $x(t)$ is not Markovian since, in addition to $x(t^-)$, one needs $C(t^-)$—which determined whether the potential is on or off—to predict $x(t^+)$. Also, $C(t)$ is clearly non-Markovian, since it is determined by the value of $x(t)$ at the time of the last measurement. Finally, the joint process (x, C) is Markovian because $(x(t^-), C(t^-))$ univocally determines $(x(t)^+, C(t^+))$, for all t^-, t^+, as explained in the text.

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