A spectrum of complexity uncovers Dunbar's number and other leaps in social structure

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How do societies become more complex? Are there specific scales at which they are reorganized into emergent entities? Social dynamics are shaped by each person's actions, as well as by collective trends that emerge when individuals are brought together. Features like population size, polarization, cohesion, or hierarchy add nuance and complexity to social structure, and might be present, or not, for societies of different sizes. Here we show that, while societal complexity increases monotonically with population, there are specific scales at which complexity builds up faster, one of them, similar to Dunbar's number (an estimation of the number of meaningful relationships that individuals can sustain). We have observed this by measuring, as a probe across populations of varied sizes, the sociolinguistic process that has unfolded over decades within the Spanish region of Galicia. For this, we have developed a methodological tool (social complexity spectrum, Fig 1), inspired by theoretical considerations about dynamics on complex networks, that could be applied in further study cases.

In this paper, published in Chaos, Solitons and Fractals [1] we have put forward the *social complexity spectrum*, a computational tool to study how certain social dynamics change as a function of the size of the interaction network within which they happen. One of our hypotheses is that the sociolinguistic dynamics of Galician and Castilian Spanish coexistence proceed faster when the web of interactions between speakers is simpler. Our second hypothesis is that population centers with more inhabitants foster more complex social networks. All our empirical results are consistent with what we might expect if these hypotheses hold true.

Our method quantifies how larger population centers are, indeed, on average, more complex than smaller ones. This is indicated by sustained negative correlations between the speed of the sociolinguistic dynamics and our measure of urbanity. Additionally, singular points at which correlation becomes saliently more negative suggest prominent population sizes at which social complexity builds up more rapidly.

One of these singular scales is observed when considering a linear relationship between dynamic rates and urbanity, $c \propto \alpha u$ (with $\alpha < 0$ a regression coefficient). This outstanding scale falls near the threshold of 5 000 inhabitants. Anecdotally, this is also known as Plato's number—Plato identified 5 040 as an ideal number of citizens in a Polis. Another singular scale appears when trying a power-law relationship. This prominent scale ($\hat{\theta}_u^{\dagger} = 183$ inhabitants, close to Dunbar's number) is the most salient one throughout all our spectrograms.

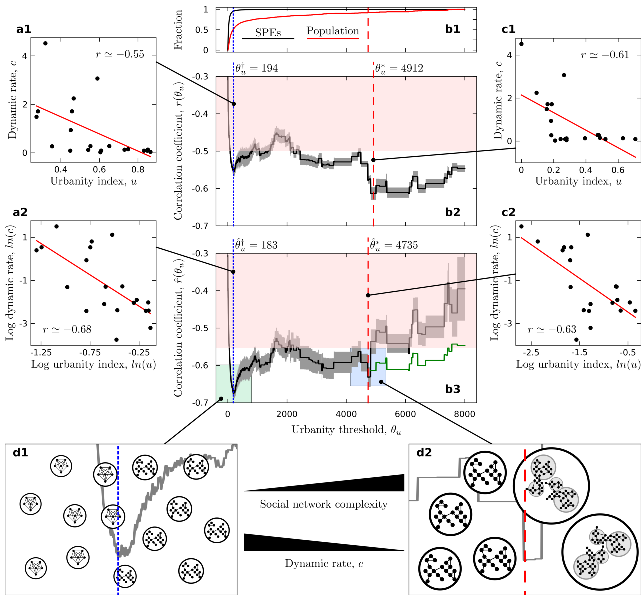

Fig. 1. **Social complexity spectra.** **a** Examples of correlations between urban indexes and dynamic rates, taking θ_u as the local minima in the prominent dip at low population sizes. The red plot marks the fraction of Galician population living in SPEs sized less than θ_u. This panel shows that nothing suspicious, which could trivially explain the properties of our spectra, happens at the outstanding scales (marked with vertical lines). **b2** Social complexity spectrum (black curve) assuming the straightforward, linear relationship $c \propto u$. Standard deviation of the spectrum (gray shading) was estimated by 10-fold jackknifing. **b3** Same for the logarithmic spectrum, which assumes $c \propto u^{\beta}$. **c** Same as in **a**, but for the spectral dip at large population size. **d** Cartoons illustrating the main implications of our hypotheses: A scale at which social complexity builds up very quickly segregates simpler from more complex social networks. Dynamics being slower in more complex networks would result in a good, negative correlation between dynamics rates within a region and that region's *urbanity*. We portray two distinct jumps in complexity to suggest that the social complexity buildup at around $\hat{\theta}_u^{\dagger}$ (**d1**) is likely very different from the one around θ_u^{*} (**d2**).

[1] Martín Saavedra, Jorge Mira, Alberto P. Muñuzuri and Luís F. Seoane *A spectrum of complexity uncovers Dunbar's number and other leaps in social structure*. Chaos, Solitons & Fractals, **170, 113389, 2023.**
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