Entropy-Based Early Detection of Critical Transitions in Spatial Vegetation Fields

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In semi-arid regions, vegetated ecosystems can display abrupt and unexpected changes, i.e., transitions to different states, due to drifting or time-varying parameters, with severe consequences for the ecosystem and the communities depending on it. Despite intensive research, the early identification of an approaching critical point from observations is still an open challenge. Many data analysis techniques have been proposed, but their performance depends on the system and on the characteristics of the observed data (the resolution, the level of noise, the existence of unobserved variables, etc.). Here, we propose an entropy-based approach to identify an upcoming transition in spatio-temporal data. We apply this approach to observational vegetation data and simulations from two dynamical models of vegetation dynamics to infer the arrival of an abrupt shift to an arid state.

Permutation entropy (PE) is a popular complexity measure for time-series analysis [1], as it is very simple to implement and robust to noise. It has also been adapted to the analysis of two-dimensional (2D) images by defining 2D ordinal patterns (2D-OPs) [2], a technique that is useful to characterize the complexity of simulated cardiac arrhythmia data [3], as well as statistical properties of textures in images [4].

Formally, spatial PE is defined as the normalized Shannon entropy:

$$H = -\frac{1}{\log M} \sum_{i}^{M} p_i \log p_i, \qquad (1)$$

where p_i are the probabilities of the 2D-OPs and M is the number of possible 2D-OPs. In the case of a continuous field, if the 2D-OPs are calculated with rectangles of X×Y pixels, then we have $M = (XY)!$. In the case of binary fields, instead, we have $M = 2^{XY}$. To examine properties at different spatial scales, one can use a lag and, in this case, the 2D-OPs are formed by non-neighboring data points.

For this study we use satellite tree cover data from the Moderate Resolution Imaging Spectroradiometer (MODIS) at 250m resolution (MOD44B) [5]. Each pixel represents a continuous variable: the proportion of tree coverage in a 250m × 250m patch. We selected transects displaying tree coverage bistability with respect to rainfall. The transects have 200×4800 pixels; pixels occupied by rivers or water bodies have been disregarded in the calculations [6].

Figure 1 displays the results of the analysis of the first transect among those selected. The average tree cover shows two clearly different states that overlap in a bistability region, where the rainfall ranges from 2900 to 3100 mm/year. Starting in the upper branch, when the mean annual rainfall decreases, both entropy and spatial correlation decrease, with the entropy rising back right before the transition. The rise of the entropy before the transition seems to be a robust indicator of the approaching tipping point as it occurs in the six transects studied, as well as in a high-resolution transect (3m × 3m) and in spatially extended models displaying critical slowing down [6] (not shown).

Like other spatial early-warning indicators, the spatial permutation entropy does not need a time series of the system dynamics, and it is suited for spatially extended systems evolving on long time scales, like vegetation plots.

Our results suggest that the spatial PE can be a good indicator of an abrupt shift in vegetation, and we believe that it has promising applications in the remote monitoring of ecosystems [6].

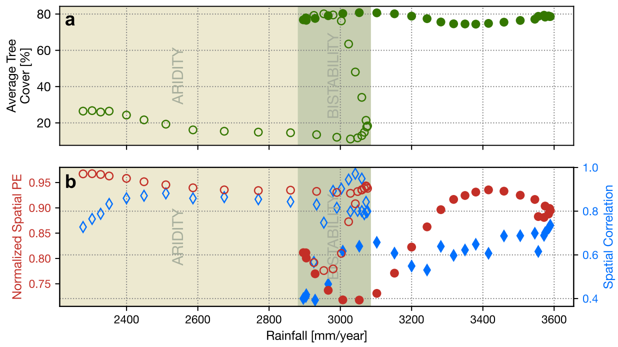

Fig. 1. Analysis of satellite vegetation data (transect 1). **a.** Average tree cover as a function of the mean annual rainfall; solid and empty symbols are used to differentiate the upper and lower branches. **b.** Spatial entropy (red) and spatial correlation (blue) of the tree cover field. These quantities have been computed using a 50km-wide window and a 12.5km step.

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