The simplest complex dynamics under friction

Martin Maza-Cuello[1], and Diego Maza[1]
[1] Departamento de Física y Matemática Aplicada, Facultad de Ciencias, Universidad de Navarra, E-31080 Pamplona, Spain.

Under its seemingly trivial nature, the dynamics of a mass over an oscillating, horizontal surface hides a rich set of non-linear behaviors in the presence of dry friction and/or viscous forces, where friction may become a driving force [1].

In this study, we present experimental measurements of the dynamics of a free, rigid body —a "walker"— over a base that oscillates sinusoidally. The setup is shown in Fig. 1. Even in this fundamental configuration, at least three dynamic regimes can be found depending on the maximum acceleration and the friction coefficients between walker and surface: i) stick, ii) stick-slip, and iii) no-stick (continuous slipping). The first transition is characterized by the crossover from static to dynamic friction, while the second one arises from a dynamical constraint on the time intervals over which the walker may slip.

Although the walker performs a periodic motion with the same frequency as the base, the dynamics is characterized by an amplitude A_W smaller than that of the base, together with a time lag τ between base and walker maxima, cf. Fig. 2.

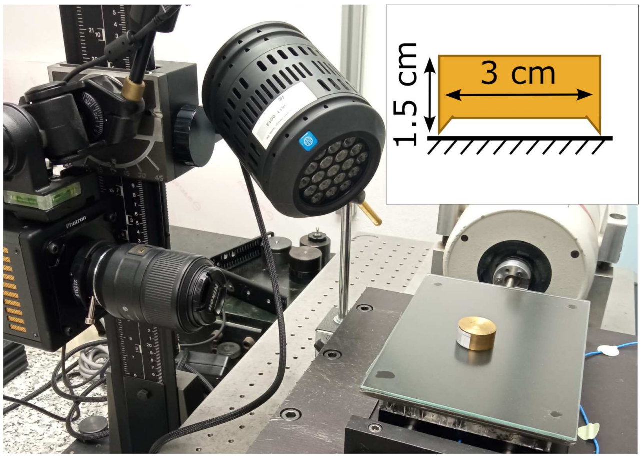

Fig. 1. Experimental setup: a walker is free to move on top of an oscillating base. The inset shows the relevant dimensions of the walker. The contact between walker and base is a ring with width $\sim$ 0.3mm. For the particular case of the image, the walker is made out of brass and the surface is frosted glass.

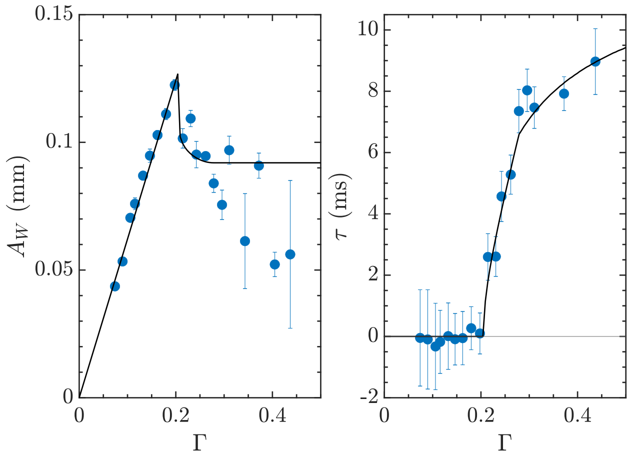

Fig. 2. Amplitude A_W of the walker's oscillations and time lag τ between the maxima of the walker and the base, for a brass walker over a quartz surface at 57 °C, as a function of the dimensionless acceleration $\Gamma \equiv A\omega^2/g$ (A: base amplitude, ω: frequency of the base, g: gravitational acceleration). Points are data averaged over 20 periods, black lines are fits to a minimal model with $\mu_s = 0.21$ and $\mu = 0.12$.

These quantities can be theoretically described from a minimal model of dry friction where the only fitting parameters are the static and dynamic friction coefficients, denoted μ_s and μ respectively. As a result, the latter can be quantified from the measurement of the walker's dynamics, allowing us to obtain values of μ_s and μ for several combinations of materials.

---

[1] M. Nicolas, *A comprehensive study on the behavior of a rigid block on an oscillating ground with friction, elastic and viscous forces*, Inter. J. Non-Lin. Mech. **93** (2017) 21–29.