Optimising search processes through heterogeneous stochastic resetting

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Stochastic resetting was introduced as a mechanism in the context of Brownian search processes [1, 2].At certain random times, the considered system restarts its natural dynamics, forgetting the past and beginning to search again until it reaches the target—see Fig. 1 for an example of a single realisation. Specifically, working on the simplest case (one-dimensional free diffusion process between resets), it is proved that the average time to hit a fixed target, also known as the mean-first passage time (MFPT), becomes finite when a Poissonian reset process with constant rate r is incorporated.

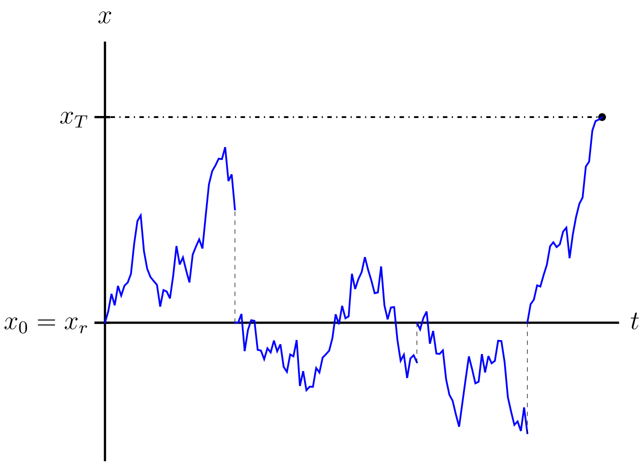

Fig. 1. Single trajectory of a free Brownian particle under the effect of stochastic resetting. The positions x_0, x_r and x_T correspond to the initial, reset and target position, respectively.

The ability of making the MFPT finite is an appealing property that has arisen the interest of the statistical physics community, since an optimisation problem appears—the MFPT diverges in both the limits of no resetting ($r = 0$) and of infinitely frequent resetting ($r = \infty$) [1]. Furthermore, broadly speaking, search processes with stochastic resetting have many applications in a wide range of contexts beyond theoretical physics, such as animal foraging, site-binding search of biomolecules of chemical reactions [2].

Previous works about stochastic resetting resetting usually considers a target at a fixed position for all realisations of the search process, *i.e* a single target remains at the same position for all realisations of the search process. Nevertheless, from a fundamental point of view, the searcher (for instance, an animal [3]) does not know where its objective (food) is, *i.e* there is some degree of randomness in the search process that renders every instance thereof unique. It is then natural to wonder how the information about the target position distribution (disorder) can guide a searcher

In this work, we address search processes with quenched (or static) disorder, *i.e.* the position of the target follows a certain static spatial distribution $p_T(x_T)$. This raises two questions: how does disorder affect the search process? Is it possible to define clever strategies for a global optimisation of MFPT, e.g. find an optimal space-dependent reset $r(x)$ given the target position distribution $p_T(x_T)$? This questions define an optimal control problem and its solution opens the door to new strategies for search processes of Brownian particles [4].

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[4] G. Garca-Valladares, C. A. Plata, A. Prados and A. Manacorda *Optimal resetting strategies for search processes in heterogeneous environments*, arXiv:2305.14245 (*submitted to New J. Phys.)*.