Numerical evidence for a phase transition in the six-dimensional Ising spin glass on a field

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Spin glasses are disordered magnetic systems that exhibit randomness and frustration, making them an example not only of glassy systems, but also of complex systems more broadly.

However, despite considerable effort, the effect of an external magnetic field ($h \neq 0$) on finite-dimensional systems is not yet fully understood. It is unclear whether there is a phase transition to a spin glass phase at all. Some results from droplet theory suggest that there is no phase transition, regardless of the finite dimension of the system [1]. Other droplet model supporters believe that it only exists above six dimensions [2]. Numerical simulations are inconclusive for $D = 3$ [3] and suggest a positive answer for $D = 4$ [4]. Further confusion is added by the failure of the standard field-theoretical approach, as no stable one-loop perturbative fixed point on the renormalization group has been found below or at $D = 6$. Finally, a two-loop computation does find a non-trivial stable fixed point at $D = 6$ [5], the Gaussian one still being unstable. This non-trivial fixed-point would lie in the non-perturbative region, being unclear whether or not the fixed point would survive beyond the two-loop computation.

Even within the field-theoretical framework, the value of the upper critical dimension D_u^h for spin glasses in a field is an open question. The classical result from replica field theory states $D_u^h = 6$, but a recent work suggests a different value, $D_u^h = 8$ [6].

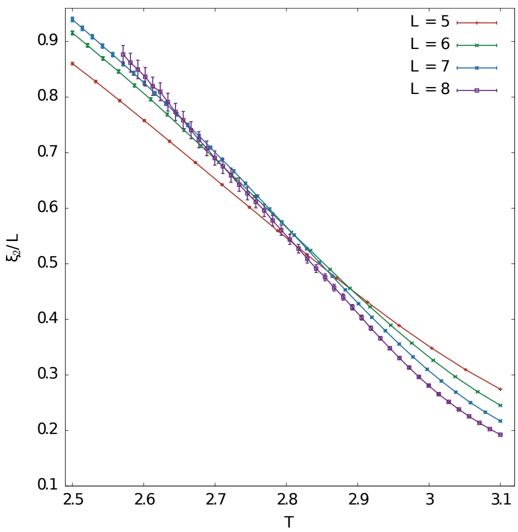

Fig. 1. Behavior of the correlation length in lattice units as a function of the temperature for the different simulated lattices. The crossing points indicate the presence of a phase transition in a field ($h = 0.075$).

In this work, we present results from massive numerical simulations of the Ising spin glass in six dimensions in a field using advanced computational and statistical techniques, such as Multi-Spin Coding, Parallel Tempering, and a thermalization protocol based on the monitoring of the temperature random walk.

In Fig. 1 we show the behavior of the correlation length, ξ_2/L in lattice units, as a function of temperature for the four lattice sizes simulated. This behavior, with crossing points, marks the presence of phase transition. Furthermore, this conclusion is supported with the study of a cumulant which avoid the use of the zero modes in its definition (see Fig. 2).

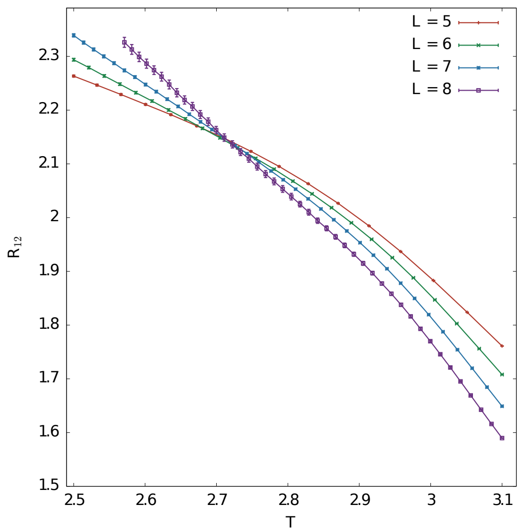

Fig. 2. Behavior of the R_{12} cumulant (defined avoiding the zero modes) as a function of the temperature for the different simulated lattices. The crossing points show the presence of a phase transition.

Finally, we show that the phase transition is correctly described by a replica-symmetric Hamiltonian and that the effective critical exponents are compatible with the Gaussian ones, suggesting that the upper critical dimension is $D_{\mathrm{u}}^h = 6$.[7]

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