Infering the connectivity of complex networks using ordinal transition methods

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Time series analysis has garnered significant research attention in recent decades. However, the exponential growth in data generation from various social, technological, and natural sources observed in the last years has posed a challenge for researchers seeking to extract valuable information from these datasets. Among the set of new tools developed for this purpose, the ordinal methods derived from the seminal work of Bandt and Pompe [1] have emerged as particularly intriguing for analyzing correlated data series [2, 3, 4]. However, using these methods to understand the information exchange in networks of dynamical systems and uncover the interplay between dynamics and structure during the synchronization process remains relatively unexplored. Here, we compare the ordinal permutation entropy, a standard complexity measure in the literature, and the entropy of the ordinal transition probability matrix that describes the transitions between the ordinal patterns derived from a time series.

Our findings, using networks of coupled chaotic Rössler systems [5], demonstrate that ordinal transition methods outperform conventional ordinal patterns' statistics when it comes to detecting subtle dynamical changes and discriminating nodes based on their topological roles [6]. In particular, to assess the methods validity in a more realistic environment with available ground truth structural information, we analysed the experimental datasets of networks of nonlinear electronic circuits downloadable in Ref. [7]. These datasets comprise the time series of the output voltage of $N = 28$ electronic circuits coupled in 20 different network configurations and monitored along their synchronization process for 100 coupling levels, ranging from disconnection (isolated nodes) to values producing a network state of complete synchrony. These experimental datasets provide the ideal testbed for our inference method and to predict the circuits connectivity by means of the network permutation entropy of each timeseries circuit as shown in Fig. 1.

These initial results illuminate new possibilities for using ordinal methods in various applications, including functional brain data analysis, power grids, mobility networks, or any arbitrary real-world time series exhibiting correlations originating from an existing underlying unknown network structure. Many methods focused on the structurefunction relationship are primarily intended to infer the detailed connectivity network, down to the level of the individual links, from time series. However, in many cases, knowledge of centrality roles alone is sufficient for designing successful interventions in the dynamics. Therefore, we anticipate that our results, which do not rely on pairwise correlations between time series, will be of particular interest in the context of functional networks and other scenarios in which the underlying structural information is inaccessible.

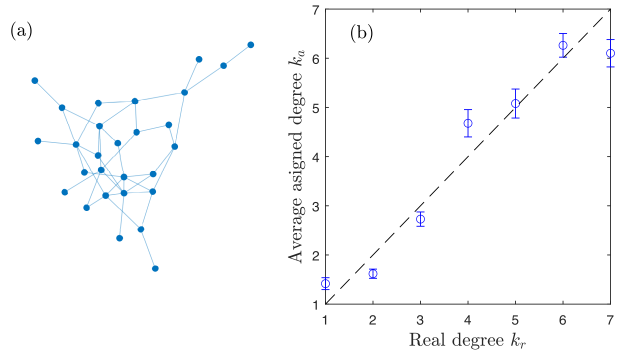

Fig. 1. Inference of the nodes' degree of networks of electronic circuits based on the network permutation entropy of the timeseries reported in Ref. [7]. (a) Structure connectivity of the electronic circuit network used as a ground truth. (b) Average assigned degree k_a versus the real degree k_r obtained when using a single network as a training reference.

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