Structural Balance in social networks: data analysis and modeling

Miguel A. González Casado[1], Andreia Sofia Teixeira[2], and Angel Sánchez[1,3]
[1]Grupo Interdisciplinar de Sistemas Complejos (GISC), Universidad Carlos III de Madrid, 28911 Leganés, Spain
[2]LASIGE, Departamento de Informática, Faculdade de Ciências, Universidade de Lisboa, Portugal
[3]Instituto de Biocomputación y Física de Sistemas Complejos (BIFI), Universidad de Zaragoza, 50018 Zaragoza, Spain

Within the network theoretical framework, a signed network is one in which links can be positive or negative, representing, for instance, friendship or enmities in personal networks. Structural Balance Theory, used to describe and understand how social networks are formed, affirms that human societies tend to avoid tensions and create balanced triangles [1], which are triangles with an even number of negative links. In this work, we unveil the role that Structural Balance (SB) plays, and which other mechanisms drive the dynamics of personal networks, in order to formulate a model that reproduces the longitudinal data based only on these local mechanisms.

First, we study the temporal evolution of a real social signed network composed by high school students, in which links between them can be either friendships, neutral or enmities. Specifically, we focus on a dataset containing 7 waves of the network (i. e., seven temporal pictures of the network between 2020 and 2023). We use the Triadic Influence (TI) [2] as a proxy measure of local SB in the network.

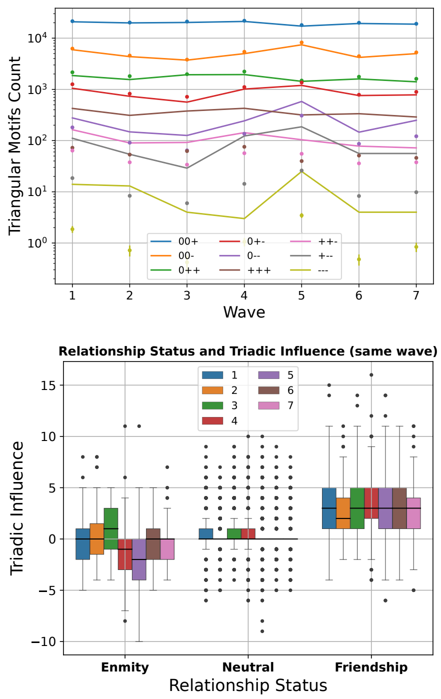

Fig. 1. **Top**: Temporal evolution of the abundance of triangular motifs. Small points represent the expected abundance of each motif in a randomized version of the same network. **Bottom**: Box Plot representing the comparison between the Relationship Status of a link and its TI.

All in all, we find that, although the rate at which links get updated is substantially high, the network remains stable around a *stationary state*, with the total number of friendships, enmities and abundances of different triangle motifs (Fig. 1) fluctuating around a stable value. The dynamics of the network appears to be driven by competing mechanisms compensating the creation and destruction of friendships and enmities. Interestingly, we observe that enmities are much more volatile than friendships, and tend to disappear faster. From a sociological perspective, friendships are maintained due to the investment of cognitive resources, and so are enmities. Nonetheless, people are not willing to invest their preciously limited amount of cognitive resources in conflicting relations, and thus they end up dissolving. Furthermore, we find that SB plays a key role in the dynamics, with the TI being a proxy of the existence of friendships (Fig. 1), and this effect is observed as well for enmities (although it is harder to detect it due to their fast disappearance rate). However, we find that enmities can also appear at random (independently from SB), something that is observed in the destruction of friendships as well. This suggests that there is a second mechanism of stochastic nature driving conflicts, competing with SB in the creation of friendships. Finally, regarding the disappearance of enmities, as we mentioned it occurs randomly and at a constant rate, suggesting that a conflict has a defined life expectancy and then it disappears, compensating for the effect of SB in the creation of enmities.

The analysis of this rich dataset leads us to propose a simple model able to reproduce the observed behavior based on three simple mechanisms: Structural Balance (driving the creation of friendships and, partially, enmities), random conflict (driving the destruction of friendships and, partially, the creation of enmities) and the life-expectancy of enmities (driving their disappearance). We calibrate the parameters of our model using real data. Interestingly, we found that the model must include the Dunbar structure of social relationships [3] to capture the observed phenomenology. With the optimal parameter values, the model is able to reproduce the observed temporal evolution of the network to a great extent, even reproducing some cascading effects in the creation and destruction of friendships and enmities we find in the data. Furthermore, we assess the generality of the model by applying it to datasets obtained in different schools.

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