The role of averaging in multidimensional symbolic dynamics

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Different data analysis tools have been developed, and applied with different rates of success, to deal with sets of time series (TS) that represent the evolution in time of some variable in different points of space (or channels), such as electroencephalograms (EEG), satellite data, or functional magnetic resonance imaging (f–MRI). One of these successful tools is Permutation Entropy (PE) [3]. Originally, PE was developed to analyse one-dimensional TS, constructing symbols from the relative magnitude of the TS at different times [1]. Different strategies have been proposed to generalise PE to deal with higher order TS, such as using all the available data across channels to calculate a single value of PE. This operation is known as pooling, and the value of PE calculated by pooling is called Pooled PE (PPE) [2]. However, many works use some average, either in time or space, to generalise PE to their data without explicitly mentioning what is the consequence of this operation, from a information theory perspective, or how the value of PE reported relates to PPE. Here, we extend the work done by Keller & Lauffer [2], who introduced PPE and studied its relation with averaged PE, by exchanging the role of space and time. As Keller and Lauffer obtained a measure of heterogeneity in the symbol distribution of different channels, we report here a new measure of the heterogeneity in time of the symbol distribution in space. Such measure can be used to detect non-stationarity in the system. We discuss what kind of structures in the data would each measure be able to detect, and applied them in synthetic systems close to bifurcation, coding them in both usual PE symbols and spatial PE (SPE) symbols [4]. We found that, for these synthetic systems, these measures of heterogeneity are able to anticipate the bifurcation, and can successfully detect non-stationary TS.

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