Making rare events typical in chaotic maps

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Dynamical fluctuations or rare events associated with atypical trajectories in chaotic maps due to specific initial conditions can be very relevant, as the may lead to stability islands or regions in phase space with other features of interest. Yet, finding such initial conditions is a daunting task precisely because of the chaotic nature of the system. In this work, we circumvent this problem by proposing a framework for finding an effective topologically-conjugate map whose typical trajectories correspond to atypical ones of the original map. This is illustrated by means of examples which focus on counterbalancing the instability of fixed points and periodic orbits, as well as on the characterization of a dynamical phase transition involving the finite-time Lyapunov exponent. The procedure parallels that of the application of the generalized Doob transform in the stochastic dynamics of Markov chains, diffusive processes and open quantum systems, which in each case results in a new process having the prescribed statistics in its stationary state. This work thus brings chaotic maps into the increasing family of systems whose rare fluctuations can be characterized and controlled by means of a large-deviation formalism.

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