Impact of overlapness on dynamical systems with higher-order interactions

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Higher-order structures are the most widely used framework to embody group interactions, and the influence of these structures on dynamical processes has been extensively addressed in recent years. In this panel we propose a metric which characterizes the overlapping between groups, i.e. hyperedges, from the perspective of the individual, i.e. node.

The surrounding environment of a node is determined by the hyperedges to which it is connected. In the case of pairwise interactions, each of them accounts for a different neighbour. However, in presence of higher order interactions, a node may have coincident nodes in different hyperedges. Thus, we introduce, for each node i, the *local overlapness* $T_i^{(m)}$ of the $m-$order hyperedges. This metric measures the normalized difference between the number of unique neighbours $\mathcal{S}_i^{(m)}$ the node has, and the minimum number of neighbours $\mathcal{S}_i^{(m),-}$ the node must have, given its value of connectivity $k_i^{(m)}$. The expression reads

$$T_i^{(m)} = 1 - \frac{\mathcal{S}_i^{(m)} - \mathcal{S}_i^{(m),-}}{\mathcal{S}_i^{(m),+} - \mathcal{S}_i^{(m),-}}, \tag{1}$$

where $\mathcal{S}_i^{(m),+}$ is the maximum number of unique neighbors a node must have given its generalized degree $k_i^{(m)}$. In the definition, we consider the substraction to 1 in order to set $T_i^{(m)} = 0$ when there is no local overlapness at all, and $T_i^{(m)} = 1$ for the maximum overlapping scenario. Once defined the set of local metrics, we introduce the *global overlapness* as the weighted mean $\mathbf{T}^{(m)}$. In this panel we are going to restrict to just to pairs (1-hyperedges) and triplets (2-hyperedges), and thus our control parameter is $\mathbf{T} = \mathbf{T}^{(2)}$. We have studied the relevancy of this metric on real data sets, obtaining a broad range of metric values.

Henceforth, we focus on its impact on contagion and synchronization dynamics by means of a synthetic structure. For the shake of simplicity, as contagion dynamics we consider an Higher-order Susceptible-Infected-Susceptible (HO SIS) compartmental model. According to it, the contagion may occur through both pairs and triplets with two distinct transmission ratios rescalated by the recovery rate, $\lambda^{(1)}$ and $\lambda^{(2)}$ respectively. On the other hand, as synchronization dynamics we contemplate the higher-order Kuramoto model, where oscillators asociated to each node evolve according to a individual natural frequency and to the coupling with pairs and triplets. The strength of the former is named $\lambda^{(1)}$ and the strength of the latter is $\lambda^{(2)}$.

Our results show that the order of both contagion and synchronization transitions changes depending on the level of overlapness. We characterize in Fig 1 and Fig 2 the phase diagrams of both dynamics as a function of the strength of the pairwise interactions and the overlapness. The similarity between the diagrams indicates universality in our findings: when higher-order structures are not locally congregated and connect all regions of the structure, explosiveness arises. However, in case they just enforce locally several nodes subsets, the phenomenology changes and a second order transition is obtained. These reasoning are validated by microscopic analysis based on effective frequencies and local synchronization on Kuramoto dynamics.

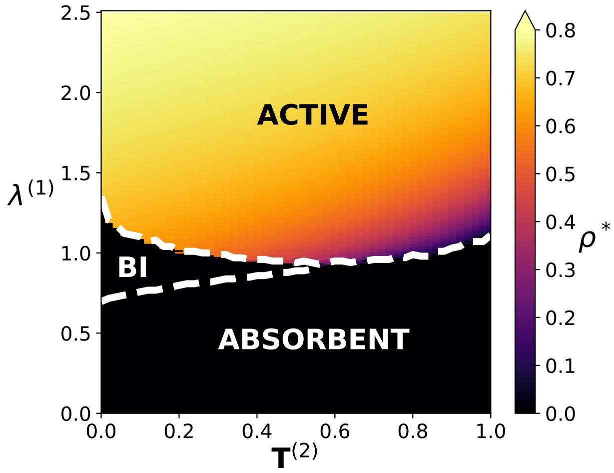

Fig. 1. SIS dynamics phase diagram is shown for $\lambda_T = 3$. Three phases emerge: absorbent phase where epidemic dies out, active phase where an endemic stationary state is reached, and a bi-stable region where the outcome depends on the initial conditions.

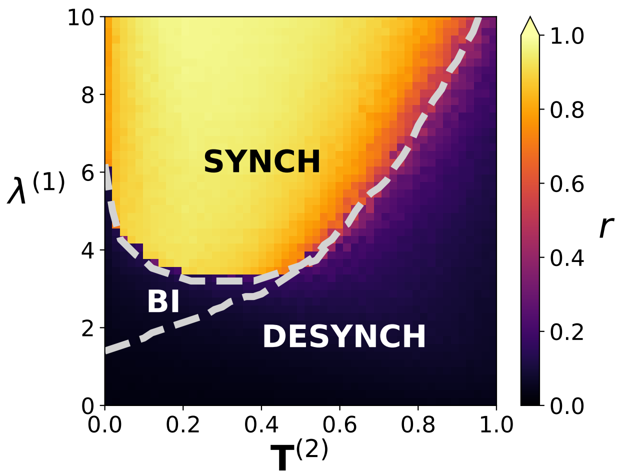

Fig. 2. Kuramoto dynamics phase diagram is shown for $2-$hyperedges coupling $\lambda^{(2)} = 3$. Three regions emerge: synchronization where all oscillators are locked, desynchronization where are unlocked, and a bi-stable regime.