Partisan Voter Model: Stochastic description and noise-induced transitions

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The paradigmatic *voter model* (VM) [1] is a stochastic binary state model of opinion formation in a population of interacting agents that imitate each other at random. The imitation mechanism accounts for the herding phenomena observed in many social systems. The Partisan Voter Model (PVM) [2] is a variation of the VM in which every agent has a fixed preference for one of the two states.

We consider here a mean field version of the PVM in a fully connected network. Voter $i \in [1, N]$ holds a binary state variable $s_i \in \{-1, +1\}$ and a preference for one of the two states. We consider that a fraction q of agents prefer to be in the state $+1$, and the rest are neutral. The preference is quantified with the parameter $\varepsilon \in [0, 1]$, which has been chosen to be the same for all voters which show a preference. An agent will copy another agent's agent with probability $\frac{1+\varepsilon}{2}$ when this state coincides with its preference, or with probability $\frac{1-\varepsilon}{2}$ otherwise. As in the VM, the PVM presents two absorbing states corresponding to the two configurations in which all agents are in the same state, either $+1$ or -1.

Due to the preference of the voters, two variables are needed for the macroscopic description of the system. They have been chosen to be the sum Σ and the difference Δ of the densities of agents that are in their preferred state. An analysis of the dynamical equations shows that, for small ε, Σ is a fast dynamical variable while Δ is a slow one. This allows us to apply an adiabatic elimination technique where, at long time scales, we describe approximately the behavior of the system by slaving the dynamics of the fast variable, Σ, to the slow one, Δ.

With this reduced model, we have been able to determine analytically the following observables:

1. Exit probability $P_q(\Delta)$, defined as the probability of reaching the absorbing state $+1$ starting from an initial condition $\Delta_0 = \Delta$. In the thermodynamic limit it tends to 1 (resp. 0) if $q > \frac{1}{2}$ (resp. $q < \frac{1}{2}$). In the symmetric case $q = \frac{1}{2}$ it is equiprobable to end up in any of the absorbing states.
2. Fixation time $\tau(\Delta)$, defined as the average time to reach any consensus state starting from an initial condition $\Delta_0 = \Delta$ and whose expression in the thermodynamic limit becomes

$$\lim_{N\to\infty} \tau(\Delta) = \begin{cases} \exp N, & \text{if} \quad |2q-1| < \varepsilon, \\ \log N, & \text{othewise}, \end{cases} \tag{1}$$

3. Quasi-stationary distribution $P_{\text{qst}}(\Delta)$, defined as the conditioned probability of the process to non-extinction, which captures the long term behavior of a process that has not yet reached the absorbing state. It exists only if $|2q-1| < \varepsilon$.

All the analytical results agree with simulations of the full, two variable, model.

Similarly to the Noisy Voter Model (NVM) [3], we introduce the Noisy Partisan Voter Model (NPVM) which includes idiosyncratic behavior with spontaneous changes of state. As in the NVM there is a competition between the herding, with rate h, and the idiosyncratic behavior, with rate a. We focus our work on the study of the stationary probability distribution examining the alterations of the noise-induced transition of the NVM. In the NVM the system switches from a bimodal to an unimodal distribution passing through a flat distribution for a critical value of the ratio $\frac{a}{h}$. In the NPVM there is a rich phase diagram with continuous and discontinuous noise-induced transitions depending on the values of q, ε and $\frac{a}{h}$. In Fig. 1, we display the parameter diagram $\varepsilon - \frac{a}{h}$ for $q = \frac{1}{2}$. For general $\varepsilon > 0$, the system exhibits three distinct regimes characterized by a different shape of the stationary distribution. In regions I and II the steady state is trimodal. The absolute maximum changes abruptly when crossing $\left(\frac{a}{h}\right)_c$. The system goes from two maxima at the borders to one central maximum. Along the line $\left(\frac{a}{h}\right)_c^*$, the lateral maxima disappear and in region III the distribution is unimodal. Additionally, the inset of Fig. 1 demonstrates that both transition lines decrease its value as the system size increases, indicating that they are finite-size transitions. In the thermodynamic limit $N \to \infty$, regions I and II disappear and the system only presents a unimodal distribution located at the center of the interval $\Delta = 0$.

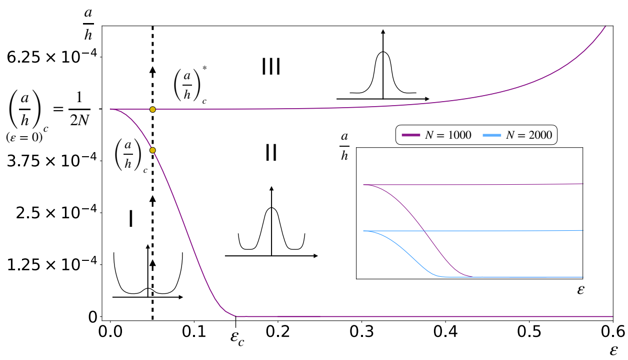

Fig. 1. Parameter diagram $\varepsilon - \frac{a}{h}$ for the different regimes of the stationary probability distribution $P_{\text{st}}(\Delta)$ for a system size $N = 1000$. Inset: Comparison of the parameter diagram $\varepsilon - \frac{a}{h}$ for system sizes: $N = 1000, 2000$ as indicated. Parameter values: $N = 1000$, $q = 0.5$, $\varepsilon = 0.05$.

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