Two diverse types of criticality in neural network models

Carles Martorell[1,2], Rubén Calvo[1,2], Miguel A Muñoz[1,2], and Alessia Annibale[3]
[1]Departamento de Electromagnetismo y Fsica de la Materia, Universidad de Granada E-18071, Granada, Spain
[2] Instituto Carlos I de Fsica Terica y Computacional, Universidad de Granada E-18071, Granada, Spain
[3]Department of Mathematics, Kings College London, SE11 6NJ London, United Kingdom

The idea that biological and artificial learning systems may extract crucial functional advantages by exploiting collective effects, in general, and critical phenomena, in particular, has been gaining more and more momentum in the last decades. At the edge of a phase transition, between order and chaos, the system can exploit the combined advantages of stability (order) and responsiveness to perturbations (chaos) without saturating and obtaining an optimal computational performance. From a biological point of view, it has been suggested that computational capabilities of brain networks stem from that delicate balance between order and disorder [1].

Recent works in neuroscience has underscored the dichotomy between two types of phase transitions in neural network dynamics: Type-I criticality, consisting on a transition to a synchronous state; and Type-II criticality, a transition to a non-synchronous and chaotic regime [2]. Both types of criticality can be characterized by the shape of the spectral distribution -the probability distribution of eigenvalues- of the adjacency matrix, which encodes the topological structure of the network. Hence, Type-I is related with the emergence of a dominant eigenvalue, also called "outlier"; while Type-II is characterized by a bulk of eigenvalues (see Figure 1).

Here, we aim to shed further light on these issues by studying one of the simplest possible neural network models, very similar to the classical model of Sompolinsky, Crisanti, and Sommers [3]. In particular, our goal is to establish a correspondence between the Ising and spin-glass transitions in statistical physics and type-I and type-II criticality in simple neural-network models.

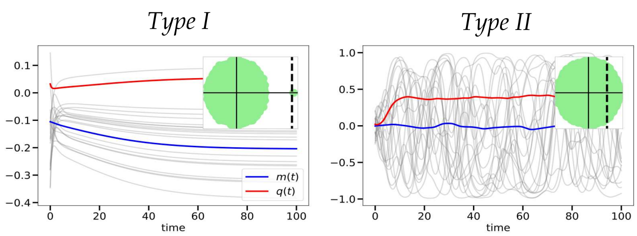

Fig. 1. **Type I vs Type II criticality.** Two different realizations for system (1) at the Ferromagnetic (F) and Spin-Glass (SG) regimes, respectively. The inset shows the spectra distribution for the adjacency matrix, gJ.

Our model consists of a fully-connected network with N neurons, each one described by its time-dependent rate, $x_i(t)$, with $i = 1, 2, ..., N$, whose dynamics obeys the following set of coupled stochastic differential equations

$$\dot{x}_i = -x_i + \tanh\left(g \sum_j J_{ij} x_j\right), \quad i = 1, \dots, N \quad (1)$$

where the synaptic weights J_{ij} are (quenched) Gaussian random variables with mean J_0 and variance J (convenient scaled to guarantee the convergence in the infinite-network-size limit). To study analytically this problem we use a Dynamical Mean Field (DMF) approach leading to a self-consistent stochastic equation for the dynamics of a representative neuron,

$$\dot{x}(t) = -x(t) + \tanh[J_0 g M(t) + \phi(t)]. \quad (2)$$

where $M(t) = \langle x(t) \rangle$ is the first moment, and $\phi(t)$ is a white-noise process with zero mean and $\langle \phi(t)\phi(t') \rangle = J^2 g^2 \langle x(t)x(t') \rangle$. This simple equation is equivalent to the well-known Sherrington-Kirkpatrick model for spin-glasses, within a replica symmetric ansatz, on which $1/g$ plays the role of the temperature. The main result is shown in Figure 2, on which the first moment (M) and the second moment (q) are plotted as a function of the parameters $\phi_1 = J_0/J$ and $\phi_2 = 1/gJ$. We proved analytically that three different regimes emerge (paramagnetic (P), ferromagnetic (F) and spin-glass (SG), by analogy), where Type-I criticality is characterized as a transition from P to F regimes, while Type-II is defined as a transition from P to SG regimes.

This work serves as an initial point to study the emergence of criticality for more sophisticated neural networks – for biological-plausible architectures: for instance, implementing the Dale's principle– by means of DMF framework.

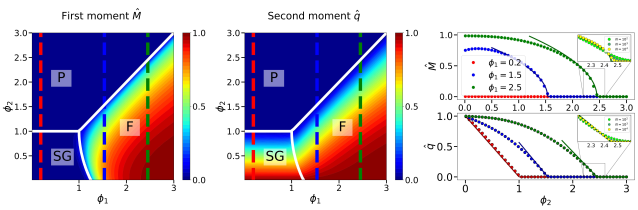

Fig. 2. **Left and middle**: Heat map of the time-averaged mean activity $\hat{M}$ (left) and mean square activity $\hat{q}$ (middle) obtained from simulations, as a function of the parameters ϕ_1, ϕ_2. The white line shows the theoretically predicted critical lines separating the paramagnetic, ferromagnetic and spin-glass phases for fixed point solutions. **Right**: Symbols show $\hat{M}$ (top) and $\hat{q}$ (bottom) obtained from simulations, versus ϕ_2. Full lines show the theoretically predicted asymptotic behaviour around the critical point. The inset shows the effect of increasing the system size.

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