

# Polarization measures in multi party elections

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Social polarization is a pervasive phenomenon that has been observed in a wide variety of contexts like elections, referenda and around controversial issues. Polarization has been traditionally studied in binary conflicts where two groups support opposite ideas.

In this work, we introduce new polarization metrics for multidimensional scenarios and develop a methodology that extracts the ideological structure of multipolar contexts from social networks. We use our framework to model multiparty democracies by considering each party as an opinion pole. If there are  $n$  opinion poles, we place each pole at the vertex of a regular simplex of dimension  $n - 1$ , being each pole at the same distance of the others.

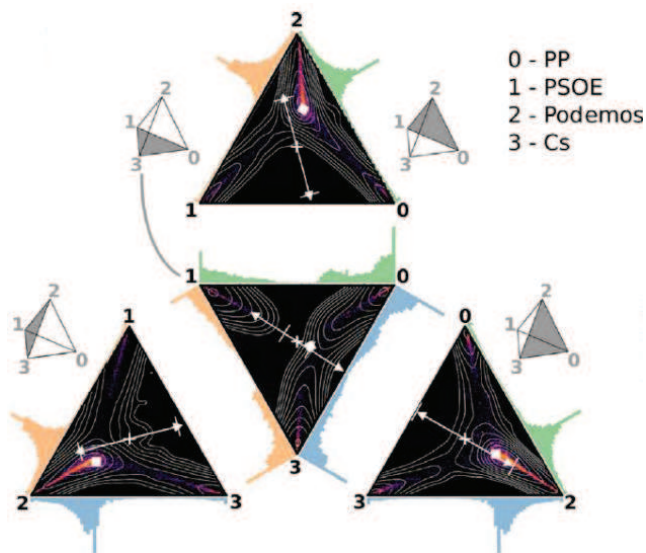


Fig. 1. Quadripolar opinion distribution of the Spanish general elections of the 20 of December, 2015.

Our multidimensional opinion inference technique is a generalization of a bipolar (one-dimensional) methodology [1] based on models of opinion dynamics. The process consists in building a network of social interactions from empirical data, identifying the opinion leaders and their respective ideological positions [2]. Then, we use the model to propagate the leaders opinions throughout the rest of the nodes. Finally, we take the models outputs (the converged opinions) as the inferred opinions of the nodes. To characterize and measure the polarization of the inferred opinion distribution we propose different metrics based on the covariance matrix [3], which is the multidimensional generalization of the variance, a quantity often adopted as a one-dimensional measure of polarization. In particular, we use the trace of the covariance matrix (the total variation) as a global measure of opinion extremeness, and its eigendecomposition to quantify pole alignment (a multipolar analogue of opinion

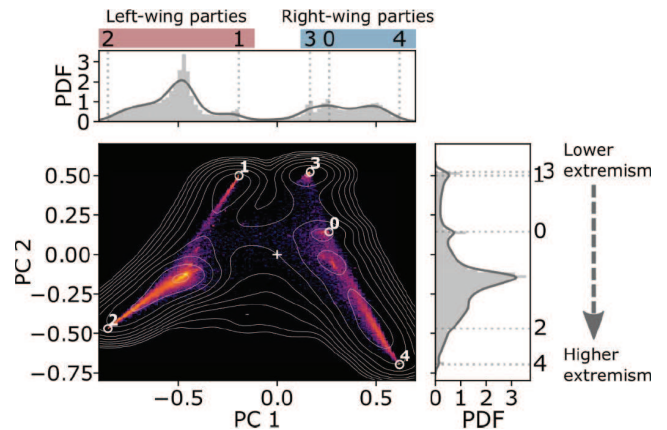


Fig. 2. Inferred ideological space of the Spanish general elections of 28 of April 2019. The 5 opinion poles are labeled from 0 to 4 and correspond to the main political parties (0-PP, 1-PSOE, 2-Podemos, 3-Cs, 4-Vox)

alignment), obtaining the direction of maximum polarization in the ideological space by principal components (PC).

We have applied this methodology to empirical Twitter data from multi-party elections: the Spanish general elections of 2015 (four poles) and 2019 (five poles).

Figure 1 shows the opinion space in the case of the 2015 Spanish general elections. Projections of the opinion distribution onto the simplex faces of the tetrahedron are shown as heat maps and contour plots. The centers of mass of the projected opinion distributions are represented as white squares and the projection of the direction of maximum polarization (PC 1), as a double headed arrow. 1D projections onto each edge of the simplex are shown on the sides of the triangles.

Figure 2 shows the two-dimensional heatmap and contour plot correspond to the projection of the inferred opinion distribution onto the first two PCs. The top and right plots respectively show the one-dimensional projection of the opinion distribution onto the first and second PCs. The parties are spontaneously sorted along the left-wing / right-wing axis in the first PC and according to their perceived extremism in the second PC.

[1] Morales AJ, Borondo J, Losada JC, Benito RM. *Measuring political polarization: Twitter shows the two sides of Venezuela*. *Chaos* **25** (3) 033114 (2015).

[2] Borondo J, Morales A, Benito R, Losada J. *Multiple leaders on a multilayer social media*. *Chaos Solitons Fractals* **72** 908 (2015).

[3] Martin-Gutierrez, S, Losada, JC, Benito, RM. *Multipolar social systems: Measuring polarization beyond dichotomous contexts*. *Chaos Solitons and Fractals* **169** 113244 (2023).