Inferring the connectivity of coupled oscillators from sequences of intervals between events

R. P. Aristides[1], G. Tirabassi[2], H. Cerdeira[1] and C. Masoller[2],
[1]Instituto de Física Teórica, Universidade Estadual Paulista, São Paulo, Brazil
[2]Departament de Fisica, Universitat Politecnica de Catalunya, Terrassa, Spain

The Kalman filter is a well-known technique to infer the parameters of a model given uncertain observations. We have recently used a nonlinear version, the unscented Kalman filter (UKF), to infer the connectivity of a small network of coupled Izhikevich neurons [1], where the links between neurons were given in terms of an adjacency matrix, A_{ij}, and their intensity was controlled by a global coupling parameter, K. Assuming that we had full information about the temporal dynamics of the different neurons [i.e., the time series $x_i(t)$ and $y_i(t)$ obtained by simulating the Izhikevich model], we have shown that UKF allows to infer the structural connectivity, i.e., to recover the matrix KA_{ij}, even if the network is directed and evolves in time.

However, usually we do not have full information of the neurons' variables, as we can only detect the spikes. Therefore, here we assume that for each neuron i, we only know the sequence of time intervals between spikes, $\{\Delta T_i(1), \Delta T_i(2), \dots\}$ (obtained by simulating coupled Izhikevich neurons, as in [1]). Then, a natural question is, can we infer the network connectivity, i.e., the matrix KA_{ij}, from the analysis of the N spike sequences of the N neurons?

To address this problem, we first obtain, from each sequence, $\{\Delta T_i(1), \Delta T_i(2), \dots\}$, a continuous instantaneous phase, $\phi_i(t)$, by simply assuming that $\phi_i(t)$ increases linearly between 0 and 2π in each ISI (Fig. 1).

Then, we use the UKF to fit the set of N phases time series, $\{\phi_1(t) \dots \phi_N(t)\}$, to N Kuramoto phase oscillators,

$$\dot{\phi_i} = \omega_i + \sum_j m_{i,j} \sin(\phi_j - \phi_i) + \sigma \xi_i. \qquad (1)$$

The last step to recover the adjacency matrix is to apply a clustering algorithm (k-means) and transform the set of $m_{i,j}$ values into a set of 0s and 1s, which are the coefficients of the recovered adjacency matrix (Fig. 2, top panel).

To evaluate the performance of the UKF approach we use the F_1 score [$F_1 = 2TP/(2TP + FN + FP)$, where TP is the number of true positives, FP, the number of false positives and FN, the number of false negatives]. We find that UKF performance is maximized for intermediate coupling strength (Fig. 2, bottom panel).

A natural next step is to test the performance of the UKF method on experimental data. We therefore used data freely available recorded from 28 electronic chaotic oscillators [2] and addressed the challenge of inferring the connectivity of sets of 3 oscillators (i.e., classify the 3 possible links as existing or nonexisting). We used the Hilbert transform to obtain the time series of the instantaneous phase of each oscillator, and finally, used UKF to fit the evolution of the phases to the Kuramoto model. Preliminary results (not shown) suggest that UKF is able to differentiate between existing and nonexisting links, in spite of the fact that the 3 oscillators are linked to other, unobserved oscillators. Ongoing efforts are devoted to develop efficient methods for inferring the connectivity of larger sets of oscillators, and to compare the performance of UKF with other well-known methods for inferring the network connectivity (cross-correlation, mutual information, etc.)

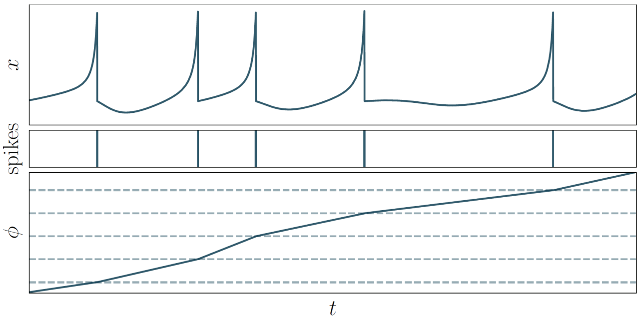

Fig. 1. (top) Simulated spike sequence, (middle) only the spike times are analyzed, (bottom) from the spike times, the time series of the instantaneous phase is obtained.

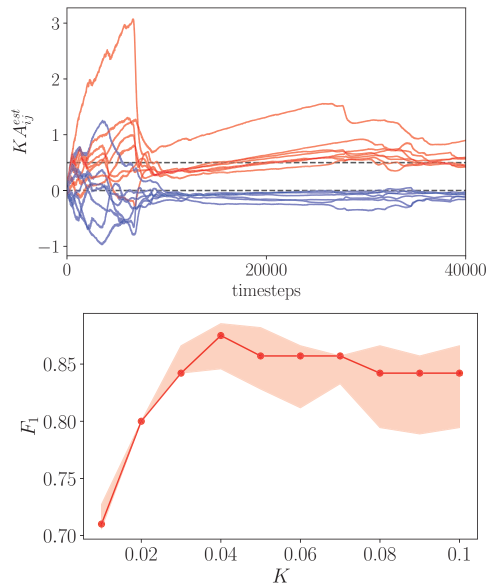

Fig. 2. (top) Estimation of the coupling coefficients. (bottom) Performance of UKF for inferring the connectivity of a network of Izhikevich neurons measured with F_1 score (15 networks were simulated, each with 6 neurons and 8 links, for each network, F_1 was averaged over 100 simulations).

[1] R. P. Aristides et al., *Parameter and coupling estimation in small groups of Izhikevichs neurons*, Chaos **33**, 043123 (2023).

[2] R. Sevilla-Escoboza and J. M. Buldu, *Synchronization of networks of chaotic oscillators: Structural and dynamical datasets*, Data in Brief **7**, 1185 (2016).