Mean-field description of a non-imitative evolutionary game

M. Aguilar-Janita [1], N. Khalil [1], I. Leyva [1 2] and I. Sendiña-Nadal [1 2]
[1]Complex Systems Group, Universidad Rey Juan Carlos, 28933 Móstoles, Madrid, Spain
[2] Center for Biomedical Technology, Universidad Politécnica de Madrid, Pozuelo de Alarcón, 28223 Madrid, Spain

The origin of cooperation in human societies remains an unresolved question. According to the principle of ”survival of the fittest,” defection should prevail over cooperation since the former provides individuals with the highest benefits.

Evolutionary Game Theory offers a theoretical framework to address this issue and that allows us to leverage techniques developed in the field of statistical physics. Within this framework, social dilemmas are formulated as two-person games, wherein each player chooses to cooperate or defect, and the game is iteratively played over multiple rounds. Mathematically, these games are described by a payoff matrix:

	C	D
C	R	S
D	T	P

(1)

Different values of the game parameters R, S, T and P describe different classical games such as *Prisoner's Dilemma*, *Harmony*, *Snowdrift*, and *Stag Hunt*. The dynamics of Evolutionary Game Theory heavily rely on how players adapt their strategies in each round of the game. Typically, the adaptation process follows imitative rules, where each player adjusts their strategy based on the outcomes of other players. The long-term stable strategies of games employing these imitative rules are significantly influenced by the population structure, characterized by the network topology connecting the players. However, recent experiments involving humans playing the *Prisoner's Dilemma* have demonstrated that the final outcome is independent of the network structure [1]. Moreover, empirical evidence suggests that humans do not take into account their neighbors' payoffs when making decisions [2]. Consequently, the community has started considering update strategies that eschew information about other players and solely depend on internal variables specific to each player [3].

In our study, we propose a novel model of non-imitative dynamics wherein agents possess an internal parameter called aspiration, denoted as m, against which they compare their payoffs. We investigate this model in a fully connected, mean-field network and determine the stationary values of the mean cooperation fraction ρ, both theoretically and through simulations based on the Gillespie algorithm. Furthermore, we introduce a mathematical framework that allows us to simplify the complexity of the parameter space to just two new parameters that encapsulate all the relevant information. These parameters are defined as follows:

$$\sigma = \frac{S-m}{|R-m|}, \tag{2}$$

$$\tau = \frac{T-m}{|P-m|}\ . \tag{3}$$

We thoroughly explore the entire parameter space for all values of R, P, S, T, and m. Additionally, we introduce an effective temperature θ which modulates the transition probabilities between cooperation and defection for each agent, introducing an stochastic component. Our investigation reveals a wide variety of stable system behaviors. Particularly, we focus on the critical behavior of the system during transitions between different states under parameter variations, including changes in the effective temperature. Furthermore, we specifically examine parameter values that correspond to classical games and draw novel conclusions within these scenarios.

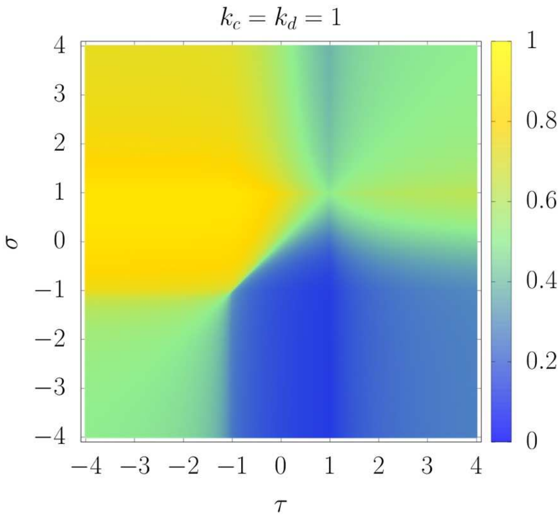

Fig. 1. Phase diagram for the two relevant parameters σ and τ for the case $R, P > m$ and an effective temperature of $\theta = 0.5$. The mean cooperation fraction ρ is codified with colors.

[1] C. Gracia-Lazaro, A. Ferrer, G. Ruiz, A. Tarancon, J. Cuesta, A. Sanchez and Y. Moreno, *Heterogeneous networks do not promote cooperation when humans play a Prisoners Dilemma*, Proc. Natl. Acad. Sci. USA **109**, 10922-20926 (2012).

[2] J. Grujić, C. Gracia-Lázaro, M. Milinski et al. *A comparative analysis of spatial Prisoner's Dilemma experiments: Conditional cooperation and payoff irrelevance*, Sci Rep **4**, 4615 (2014)

[3] G. Cimini and A. Sanchez, *How evolution affects network reciprocity in Priosioner's Dilemma*, Journal of Artificial Societies and Social Simulation **18**, 22 (2015)

[4] Nagi Khalil, I. Leyva, J. A. Almendral, and I. Sendiña-Nadal, *Deterministic and stochastic cooperation transitions in evolutionary games on networks*, Phys. Rev. E **107**, 054302 (2023)