Communicability geometry reveals antagonistic factions in signed networks

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The study of signed networks is becoming increasingly popular over the last years. One of the key challenges in this field lies in generalizing network properties while simultaneously preserving the signed nature of the edges. Specifically, the notion of distance between nodes is not clearly defined in the presence of negative connections, since the shortest-path distance ceases to be positive definite. Another significant problem in the field is the determination of the effective relation between any pair of nodes; i.e., whether they act as effective allies or adversaries. This question was answered by Harary in 1953 [1] for the case of balanced graphs (graphs where nodes can be grouped into two factions, with positive edges within the same faction and negative edges across factions). However, the question remains unsolved for the case of unbalanced graphs.

In this work, we propose a novel approach that unifies these seemingly unrelated questions through the introduction of the signed communicability, G, which is defined as the exponential of the adjacency matrix of the graph, A: $G(A) = e^{A}$. This function has been explored in depth for the case of unsigned networks [2], yet no extension to signed networks was done until this work. We have proved that, by taking into account all walks between two given nodes, the communicability function is able to measure the effective level of alliance or conflict between them. Additionally, it induces an Euclidean distance [3] that remains valid for signed graphs, overcoming the limitations of the traditional shortest path distance. Moreover, we have also proved that the communicability distance induces an embedding of the signed network into a high-dimensional hypersphere.

On the other hand, we have defined a communicability angle between nodes of a signed network, and mathematically proved that this angle provides better insights into the network's mesoscopic structure than other measures (figure 1). Guided by this insight, we devised a clustering technique to categorize nodes into opposing factions that works even when the network is unbalanced. Finally, we applied this technique to analyze voting patterns within the European Parliament, revealing that structural factions in this political entity diverge from conventional political party delineations.

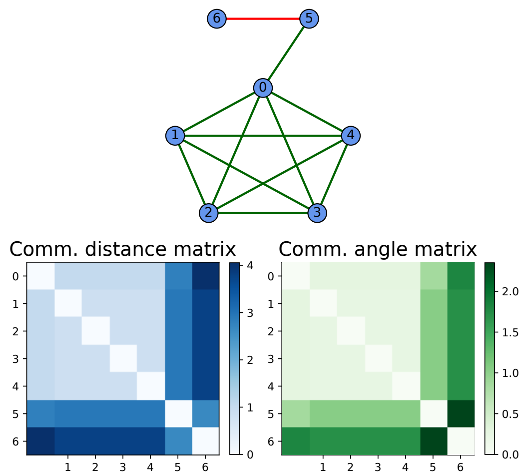

Fig. 1. Example that highlights the superiority of the communicability angle over the communicability distance (green colors represent positive edges and red colors, negative ones). The drawn graph is balanced and one of the factions is composed only by node 6, so any reliable distance measure should conclude that node 6 is farther from all other nodes compared to the distance between the remaining nodes themselves. However, the communicability distance matrix (lower left plot) situates nodes 5 and 6 closer than nodes 5 and 1. On the other hand, the communicability angle matrix (lower right plot) correctly identifies that nodes 5 and 6 are more separated than nodes 5 and 1. Consequently, the communicability angle is a more reliable distance measure.

[1] Harary, Frank. "On the notion of balance of a signed graph." Michigan Mathematical Journal 2.2 (1953): 143-146.

[2] Estrada, Ernesto, and Naomichi Hatano. "Communicability in complex networks." Physical Review E 77.3 (2008): 036111.

[3] Estrada, Ernesto. "The communicability distance in graphs." Linear Algebra and its Applications 436.11 (2012): 4317-4328.