Thermal brachistochrone: Minimizing connection times between equilibrium states in harmonically confined Brownian particles

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We are used to apply a predictive approach in physical problems: specific external conditions are considered and the evolution of the physical system is derived. However, it is not uncommon to wonder what the external protocol should be to impose in order to drive the system in a certain desired evolution. Inverse engineering techniques and, more generally, (optimal) control theory provide a robust framework to address such problems. In the last decade, the use of control theory has been proven as a productive strategy in the context of quantum mechanics, where a number of shortcut methods have been successfully applied [1].

More recently, these strategies have been brought to the field of classical statistical mechanics [2], exploiting the analogy between the mathematical descriptions of both realms. In this contribution, we present an exact shortcut within a paradigmatic model system in nonequilibrium statistical mechanics. Specifically, we consider a d-dimensional overdamped harmonically confined Brownian particle in contact with a thermal bath, in which the temperature can be externally manipulated. Therein, we obtain the shortest protocol connecting two equilibrium states at different temperatures.

The Gaussian character of the position distribution persists along the whole evolution, which reduces the problem to a system of first-order differential equations for the d variances in each dimension where the temperature appears linearly in all the equations. Direct application of Pontryagins principle leads us to obtain that the protocol giving the shortest connection time is a bang-bang protocol for the temperature between the bounds of the control, i.e., the temperature takes constant values switching between the bounds. Remarkably, the problem in the limit of spherical symmetry is singularly different from the one-dimensional case: there is an unavoidable time cost to connect systems with higher dimension as shown in Fig. 1.

We have looked into this problem of the thermal brachistochrone in detail, studying the effect of finite power to heat and cool, that is, considering finite bounds for the temperature of the thermal bath. Furthermore, the thermodynamic length and the thermodynamic cost are investigated for the optimal protocol, delving into the role of information geometry quantities within the field of stochastic thermodynamics.

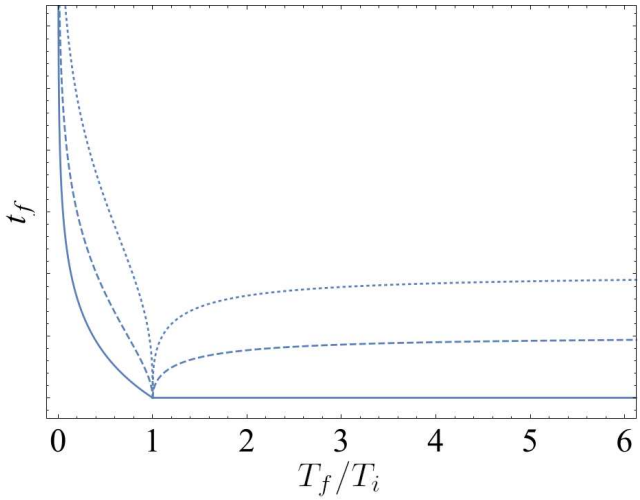

Fig. 1. Shortest time t_f for connecting two equilibrium states of a d-dimensional Brownian harmonic oscillator with a ratio T_f/T_i between the final and initial temperature for $d = 1$ (solid), $d = 2$ (dashed), and $d = 3$ (dotted). An unavoidable cost of time emerges despite the fact that the limit of spherical symmetry is assumed.

[1] D. Guéry-Odelin, A. Ruschhaupt, A. Kiely, E. Torrontegui, S. Martínez-Garaot, and J. G. Muga, *Shortcuts to adiabaticity: Concepts, methods, and applications*, Rev. Mod. Phys. **91**, 045001 (2019).

[2] D. Guéry-Odelin, C. Jarzynski, C. A. Plata, A. Prados, and E. Trizac, *Driving rapidly while remaining in control: classical shortcuts from Hamiltonian to stochastic dynamics*, Rep. Prog. Phys. **86**, 035902 (2023).

[3] A. Patrón, A. Prados, and C. A. Plata, *Thermal brachistochrone for harmonically confined Brownian particles*, Eur. Phys. J. Plus **137**, 1011 (2022).