# Random walk interpretation of kinetic theory for intruders in freely cooling and driven granular gases

E. Abad[1], R. Gómez González[2], S. B. Yuste[2], and V. Garzó[2]
[1]Departamento de Física Aplicada, Instituto de Computación Científica Avanzada (ICCAEx), Universidad de Extremadura, 06800 Mérida (Spain),
[2]Departamento de Física, Instituto de Computación Científica Avanzada (ICCAEx), Universidad de Extremadura, Avda. de Elvas s/n, 06006 Badajoz (Spain),

The Enskog kinetic theory has a longstanding tradition as a successful approach for the computation of transport properties in granular gases [1]. However, its relation with random walk approaches for the computation of such properties remains widely unexplored; the present work aims to shed further light on this relation to pave the way for future research in this field.

To illustrate the aforementioned relation, in a first stage we employ the (inelastic) Enskog-Lorentz kinetic equation in tandem with DSMC simulations to compute the mean square displacement (MSD) of intruders immersed in a granular gas of smooth inelastic hard spheres (grains). We consider the cases where the intruder-grain system includes (lacks) an interstitial molecular gas that plays the role of a thermal bath (background).

In the absence of such an interstitial fluid, there is no mechanism in this freely cooling granular gas to compensate for the continuous energy loss of the grains due to the dissipative collisions between them. Consequently, the random kicks experienced by an intruder upon collisions with grains also become less and less energetic in the course of time, and the intruder's motion is strongly slowed down with respect to the case of standard Brownian motion (as a matter of fact, the intruder's MSD exhibits a logarithmic time growth [2] instead of a linear one, and there is e.g. no longer equivalence between ensemble-averaged and time-averaged MSD among other peculiarities of this ultraslow anomalous diffusion).

We then incorporate the interstitial fluid, which has a twofold effect; on the one hand, it induces a viscous drag force acting on intruders and grains; on the other hand, it feeds both particle species with energy, this supply being modeled via a stochastic Langevin-like force defined in terms of the background temperature $T_{\mathrm{b}}$. As a result, in this driven granular gas the linear time growth of the intruder's MSD is restored (normal diffusion). However, the calculation of the associated intruder's diffusivity $D$ proves a technically challenging task which has recently been tackled in the so-called first and second Sonine approximation [3].

In a second step, we invoke an effective random walk picture of the intruder's motion to obtain an intuitive interpretation of the intricate dependence of the diffusion coefficient on the main system parameters, both with and without the interstitial fluid [2, 3]. Despite the obvious differences in the time dependence of the MSD, for a proper parameter choice one observes in both cases a nonmonotonic behaviour of the MSD at a given time $t$ as a function of the restitution coefficient $\alpha$ for grain-grain collisions (see Fig. 1 for the case with interstitial fluid and intruders mechanically equivalent to grains [selfdiffusion]). The idea underlying the random walk approach is to decompose the MSD into a product of the number of intruder-grain collisions $N(t)$ and the square of an effective mean free path $\ell_e^2$ between collisions. This latter quantity differs from the actual mean free path because of the persistence of the (strongly anisotropic) collision rules for hard spheres. While $N(t)$ increases with $\alpha$, $\ell_e^2$ decreases with this quantity because of the increased backscattering of intruder-grain collisions. The competition between the two effects then explains the aforementioned non-monotonic behaviour of the MSD. We anticipate that the overarching random walk approach presented here very likely applies to other types of driven systems lacking interstitial fluids, and in this sense the random walk interpretation is deemed to be very general.

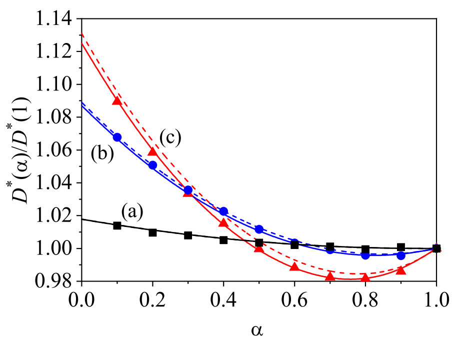


Fig. 1. Plot of the (adimensionalized) self-diffusion coefficient $D^*(\alpha)/D^*(1) \equiv D(\alpha)/D(1)$ versus the coefficient of restitution $\alpha$ for a system with (adimensionalized) background temperature $T_{\mathrm{b}}^* = 1$ and three different values of the volume fraction $\phi$ occupied by grains: (a) $\phi = 0.01$ (black lines and squares); (b) $\phi = 0.1$ (blue lines and circles); and (c) $\phi = 0.25$ (red lines and triangles). The symbols refer to the DSMC results, while the solid (dashed) lines correspond to the theoretical results obtained from the second (first) Sonine approximation. Here, $D^*(1)$ is the elastic-limit value of the self-diffusion coefficient consistently obtained in each approximation.

[1] V. Garzó, *Granular Gaseous Flows* (Springer, Cham, 2019).

[2] E. Abad, S. B. Yuste, and V. Garzó, *On the mean square displacement of intruders in freely cooling granular gases*, Granular Matter **24**, 1–19 (2022).

[3] R. Gmez-Gonzlez, E. Abad, S. B. Yuste, and V. Garzó, *Diffusion of intruders in granular suspensions: Enskog theory and random walk interpretation*, arXiv:2305.09259v1.